Stochastic FDTD Modeling of Propagation Loss due to Random Surface Roughness in Sidewalls of Optical Interconnects

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***Abstract*—The dielectric waveguide (WG) is an important building block of high-speed and high-bandwidth optical and opto-electronic interconnect networks that operate in the THz frequency regime. At the interface of Si/SiO$_2$ dielectric waveguides with width above $w = 2.5$ μm and anisotropic surface roughness, transverse electric (TE) mode surface wave propagation can experience a loss of approximately $\alpha = 2$ dB/cm; however, propagation losses increase rapidly to near $\alpha = 44$ dB/cm as the width decreases to $w = 500$ nm, due to increased interaction of surface waves and sidewall surface roughness that exhibits random distribution inherent to the manufacturing process. Previous works have developed analytic expressions for computing propagation loss in a single dielectric waveguide exhibiting random roughness. More recent works report $\alpha = 0.4$ dB/cm noting the non-trivial estimation errors in previous theoretical formulations which relied on planar approximations, and highlight the discrepancy in planar approximations vs. the 3-D Volume Current Method.**

A challenge that remains in the path of designing nanoscale optical interconnects is the dearth of efficient 3-D stochastic computational electromagnetic (CEM) models for multiple tightly coupled optical dielectric waveguides that characterize propagation loss due to random surface roughness in waveguide sidewalls. Through a series of theoretical and numerical experiments developed in the method of finite-difference time-domain (FDTD), we aim to develop stochastic CEM models to quantify propagation loss and facilitate signal & power integrity modeling & simulation of arbitrary configurations of multiple tightly-coupled waveguides, and to gain further insights into loss mechanisms due to random surface roughness in optical interconnects.

I. Introduction

A challenge in the path of designing nanoscale optical interconnects is the dearth of accurate and efficient 3-D computational electromagnetic models [1]. The state-of-the-art work by [2] reports $\alpha = 0.4$ dB/cm and highlights the non-trivial estimation errors in previous theoretical formulation [3], [4] which rely on planar approximations. The work by [2] highlights the discrepancy in planar approximations vs. the 3-D Volume Current Method (VCM) [5]–[7] which was initially developed to model symmetric structures embedded in a uniform dielectric constant.

In this paper, we verify the FDTD simulation space by correlating against known analytic solutions for wave impedance Z_w from [8]. Additionally, we replicate data from [9] for propagation loss due to surface roughness in dielectric waveguide sidewalls for future use in verification of similar models for coupled waveguides in 3-D FDTD.

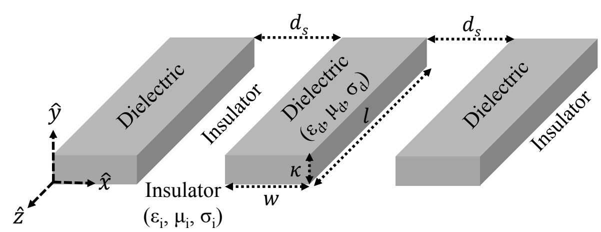

Fig. 1. Three parallel dielectric waveguides, where d_s is the edge-to-edge distance between two adjacent waveguides.

II. Formulation

A. 2-D Analytic Approximation of Z_w

The 2-D models of the dielectric waveguide developed by [8], [10] assume a transverse electric (TE) mode of propagation along the waveguide length (i.e., TE^z mode, transverse electric to $\hat{z}$), and x-invariance along the waveguide width (i.e., $\frac{\partial}{\partial x} \Rightarrow 0$) which implies an infinite width for the waveguide.

Looking from free-space insulator down towards the dielectric slab, the TE mode surface wave impedance Z_w^{-y0} may be defined by (1), and obtained by solving two non-linear equations (2) simultaneously [8].

$$Z_w^{-y0} = E_x^{0+}/H_z^{0+} = -\jmath \frac{\omega \mu_0}{\alpha_{y0}}, \tag{1}$$

where, the angular frequency $\omega = 2\pi f$ (rad/s), the cyclic frequency f (Hz), $\jmath = \sqrt{-1}$, α_{y0} is the attenuation constant in free-space along $\hat{y}$, and $\{E_x^{0+}, H_z^{0+}\}$ are the incident {electric, magnetic} field vector components along $\{\hat{x}, \hat{z}\}$; respectively.

$$(\mu_0/\mu_d)(\beta_{yd}h)\tan(\beta_{yd}h) = \alpha_{y0}h,$$
$$(\alpha_{y0}h)^2 + (\beta_{yd}h)^2 = a^2, \tag{2}$$

where, $a = \omega h \sqrt{\mu_0 \epsilon_0 (\mu_{r_d}\epsilon_{r_d} - 1)}$, and β_{yd} is the phase constant in dielectric along $\hat{y}$.

The above model is inadequate for evaluating effects of finite width on propagation loss due to wall surface roughness; however, it serves its purpose here to verify our 2-D/3-D FDTD models via correlation of surface wave impedance Z_w in ideal waveguides; as shown in Fig. 2.

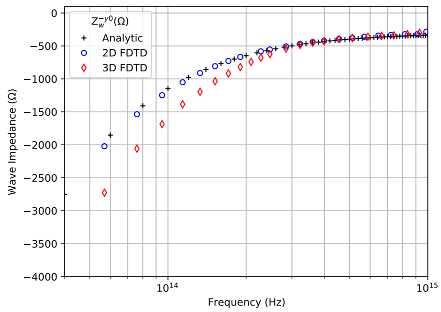

Fig. 2. Correlation of wave impedance across 2-D analytic, 2-D FDTD, and 3-D FDTD.

B. 2-D Analytic Model of Propagation Loss due to Surface Roughness

Assuming surface roughness $f(z)$ exhibits an exponential distribution, the auto-correlation function $R(u) = \sigma^2 e^{-|u|/L_c}$, where σ is the mean squared deviation of the roughness, and L_c is the correlation length. Figure 3 depicts random surface roughness of a waveguide with surface roughness on walls at $x = \pm w/2$. We orient the waveguide in the coordinate system so that it is y-invariant ($\frac{\partial}{\partial y} \Rightarrow 0$) and the roughness profile $f(z)$ varies as a 1-D function of z along the waveguide length. The dielectric slab waveguide has refractive index n_1 for its core, and n_2 for its cladding material.

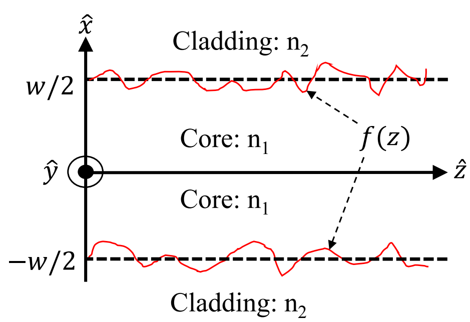

Fig. 3. Depiction of surface roughness $f(z)$ on waveguide walls, along waveguide length $2l$ [3].

Figure 4 depicts the propagation loss as a function of waveguide width, assuming infinite height. α_0 shows the results from Fig. 2 in [9], α_1 is based on β from [11] based on the solution of (2), and α_2 is based on β from the *effective index method* [12]. The $1/d^2$ curve is shown as reference for comparing losses. Figure 5 depicts the propagation loss as a function of both σ and L_c. The contours are in dB/cm and match Fig. 4 in [9].

III. Conclusion

Our 2-D/3-D FDTD models of ideal dielectric waveguides in the THz regime were validated by correlation against analytic solutions. We also replicated estimates of propagation loss, due to random surface roughness, based on 2-D planar

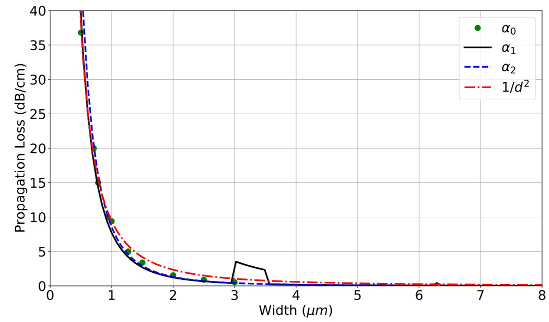

Fig. 4. Attenuation coefficient α vs. waveguide width.

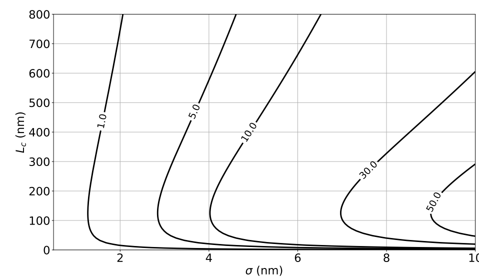

Fig. 5. α vs. L_c and σ.

approximations developed by previous investigators. Currently, work is in-progress to compute the propagation loss via 2-D/3-D FDTD for comparison against previously published data from experimental measurements and numerical simulations.

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