Enhancing Adaptive Mesh Refinement Efficiency: Adjoint-Based Error Estimation and Targeted Refinement in 3-D FEM

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This paper presents advancements in accelerated adaptive mesh refinement (AMR) for application in computational electromagnetics (CEM). In the generation of high quality discretizations we have at once two competing interests: accuracy – whether of the solution itself or some functional of the solution – and efficiency. AMR automates the necessary balance in this construction, achieving, for a given number of unknowns, a near-optimal resource allocation, or for a desired accuracy, a near-minimal number of unknowns.

Classical approaches to error estimation and adaptive refinement restrict analysis to the solution itself, usually by classifying the smoothness, and refining (or coarsening) in fixed increments. In many cases, such strategies significantly overestimate the discretization error, thereby producing inefficient meshes, or, in other cases, produce unreliable and ineffectual indicators which fail to control the error adequately. Moreover, in most cases the desired goal of solving the numerical problem is not the solution in isolation; instead, we require post-processing of the solution, such as to compute the radar cross section (RCS) or S-parameters, and therefore the AMR procedure should operate in coordination with this goal.

Existing AMR strategies often necessitate significant computational expense. By employing binary indicators for refinement, the adaptive steps neglect the degree to which a cell should be refined, relying instead on later iterations to correct poor refinement choices. While eventually producing quality discretizations, these strategies demand increasingly large AMR iteration numbers for higher accuracy.

We discuss a novel approach that combines adjoint-based *a posteriori* error estimation with *a priori* error conditions to drive intelligent adaptive mesh refinement in 3-D FEM. Highly versatile and efficient, our method produces high-quality discretizations and significantly reduces the dependence of the number of refinement iterations on the desired accuracy of the QoI by leveraging the combined error information to guide variable degrees of refinement throughout the entire discretization.

We focus our numerical demonstrations on a practical and customizable scattered field QoI. We then show the ability to rapidly refine coarse discretizations to high accuracy in few iterations and according to several user-prescribed error tolerances. Finally, we demonstrate that the discretizations significantly improve error homogenization, which indicates the efficiency of our method.