Simple Pattern Synthesis for Complicated Arrays

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***Abstract*—A simple method for approximate pattern synthesis is described. This method is applicable to arrays of any geometry and employing any element pattern, including non-uniformly spaced arrays, volumetric arrays, and arrays consisting of elements having significantly unequal embedded patterns, as is common in the presence of strong mutual coupling.**

I. Introduction

Pattern synthesis is the problem of identifying element excitations that result in an array pattern that is as close as possible in some sense to a specified pattern. Pattern synthesis methods for certain classes of arrays (e.g., uniform linear arrays of isotropic elements) producing certain types of patterns (e.g., beams with specified sidelobe characteristics) are well-known; see e.g., [1] (Sec. 10.1).

However, a general method yielding an exact solution is not available since any particular array is limited in the patterns that can be supported. For example, a uniform linear array consisting of a finite number of isotropic elements cannot normally generate a beam with lobe width less than a threshold determined by its length, nor can it generate a sector beam having exactly uniform directivity over a specified angular span. Approximate solutions are possible using the Fourier transform relationship between element excitations and array patterns; see e.g. [1] (Sec. 10.2). However traditional Fourier transform methods are not directly applicable to arrays having irregular spacings, volumetric arrays, or arrays of elements having unequal embedded patterns, as is common in the presence of strong mutual coupling.

This paper describes a simple approximate method of pattern synthesis that is applicable in these cases.

II. Method

The pattern of an N-element array is given by

$$F(\hat{\mathbf{r}}) = \sum_{n=1}^{N} b_n F_n(\hat{\mathbf{r}}) e^{+j\beta \mathbf{P}_n \cdot \hat{\mathbf{r}}} \tag{1}$$

where $\mathbf{P}_n$, $F_n(\hat{\mathbf{r}})$, and b_n are the position, pattern, and excitation, respectively, of element n; β is the phase propagation constant $2\pi/\lambda$ where λ is wavelength; and $\hat{\mathbf{r}}$ is the unit vector pointing from the origin in the direction (θ, ϕ); i.e.,

$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \sin\theta\cos\phi + \hat{\mathbf{y}} \sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta \tag{2}$$

Let $G(\hat{\mathbf{r}})$ be the specified pattern, so ideally $F(\hat{\mathbf{r}}) = G(\hat{\mathbf{r}})$. Since equivalence is generally not possible, we instead reduce this equality to N linear equations that can be solved for the b_n's. Each equation is formed by multiplying both sides by

$$\left(F_m(\hat{\mathbf{r}}) e^{+j\beta \mathbf{P}_m \cdot \hat{\mathbf{r}}}\right)^* \tag{3}$$

where the superscript "$*$" denotes the complex conjugate, and $m = 1...N$. We then integrate both sides over a sphere surrounding the array. In lieu of $G(\hat{\mathbf{r}})$, we obtain:

$$c_m = \oint G(\hat{\mathbf{r}}) F_m^*(\hat{\mathbf{r}}) e^{-j\beta \mathbf{P}_m \cdot \hat{\mathbf{r}}} \, d\Omega \tag{4}$$

where the integration is over a closed sphere bounding the array, and $d\Omega = \sin\theta \, d\theta \, d\phi$. In lieu of $F(\hat{\mathbf{r}})$, we obtain:

$$\oint F(\hat{\mathbf{r}}) F_m^*(\hat{\mathbf{r}}) e^{-j\beta \mathbf{P}_m \cdot \hat{\mathbf{r}}} \, d\Omega \tag{5}$$

After substituting Equation 1 and rearranging factors, we obtain

$$\sum_{n=1}^{N} b_n A_{mn} = c_m \tag{6}$$

where

$$A_{mn} = \oint F_n(\hat{\mathbf{r}}) F_m^*(\hat{\mathbf{r}}) e^{+j\beta(\mathbf{P}_n - \mathbf{P}_m)\cdot \hat{\mathbf{r}}} \, d\Omega \tag{7}$$

This yields a system of N equations which may be written compactly as follows:

$$\mathbf{A}\mathbf{b} = \mathbf{c} \tag{8}$$

where $\mathbf{A}$ is the $N \times N$ matrix of A_{mn}'s, $\mathbf{b}$ is the $N \times 1$ matrix of b_n's, and $\mathbf{c}$ is the $N \times 1$ matrix of c_m's. Thus, the desired excitations are given by

$$\mathbf{b} = \mathbf{A}^{-1}\mathbf{c} \tag{9}$$

In this framework, $\mathbf{A}$ serves as a compact description of the array and can be pre-computed, $\mathbf{c}$ can be interpreted as the correlation between the specified array pattern and that of each element of the array, and $\mathbf{A}^{-1}\mathbf{c}$ can be interpreted as a projection of a compact representation of the specified pattern onto the space of similarly-represented patterns that can be supported by the array.

It should be noted that the method is not minimizing the difference between $F(\hat{\mathbf{r}})$ and $G(\hat{\mathbf{r}})$ in any particular sense. On the other hand, the method is deterministic (an iterative search is not required) and, as shown in the next section, useful results are possible.

III. Demonstration

To demonstrate the method, consider the $N = 17$ element array shown in Figure 1. Each arm of the array is oriented $30°$ from the y axis. Element spacings increase linearly from 0.4λ around the center to 1.2λ at the ends. The pattern of element n in direction $\hat{\mathbf{r}}$ is $\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}_n$ when this quantity is positive, and zero otherwise, yielding a single-lobe cosine pattern. The element pointing directions $\hat{\mathbf{r}}_n$ vary linearly from broadside $(+\hat{\mathbf{x}})$ for the center element to $(\hat{\mathbf{x}} \pm \hat{\mathbf{y}})/\sqrt{2}$ for the $\pm y$ arms of the array, respectively. This non-uniformity in geometry and element patterns is sufficient to defeat most commonly-cited methods for pattern synthesis.

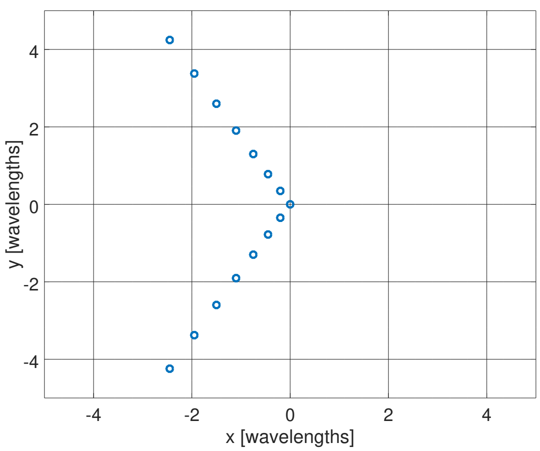

Fig. 1. Array geometry. Elements lie entirely in this ($z = 0$) plane. $\phi = 0$ (i.e., $\hat{\mathbf{x}}$) is to the right and increases counterclockwise. See text for element patterns.

Figure 2 shows the results when the specified pattern $G(\hat{\mathbf{r}})$ is the impulse function $\delta(\hat{\mathbf{r}} - \hat{\mathbf{r}}_0)$ defined as follows:

$$\delta(\hat{\mathbf{r}} - \hat{\mathbf{r}}_0) = 0, \qquad \hat{\mathbf{r}} \neq \hat{\mathbf{r}}_0 \tag{10}$$

$$\oint \delta(\hat{\mathbf{r}} - \hat{\mathbf{r}}_0) d\Omega = 1 \tag{11}$$

In this example, $\hat{\mathbf{r}}_0$ points to $(\theta_0, \phi_0) = (90°, +15°)$, which should result in a narrow beam in this direction and low sidelobes elsewhere. Shown for comparison is unconstrained "phase-only" beamforming; i.e.,

$$b_n = e^{-j\beta \mathbf{P}_n \cdot \hat{\mathbf{r}}_0} \tag{12}$$

As expected, the patterns are essentially identical around the main lobe and are qualitatively similar elsewhere.

Figure 3 shows the results when the specified pattern $G(\hat{\mathbf{r}})$ is a "sector beam" equal to 1 for $0 \leq \phi \leq 30°$ and zero otherwise. The method produces a reasonable approximation to the specified pattern. A significantly better approximation would require a larger array.

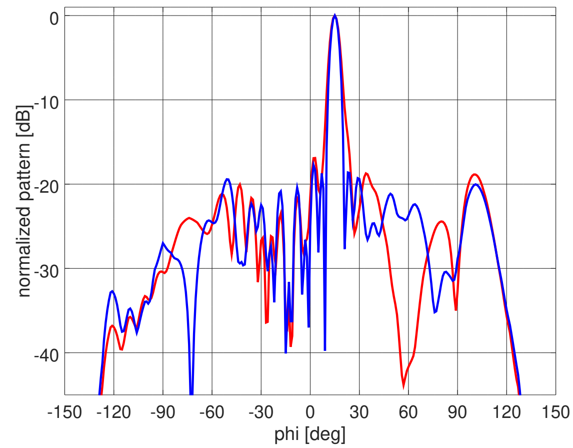

Fig. 2. Pattern from (*blue*) the proposed method for the "impulse beam" specification and (*red*) unconstrained "phase-only" beamforming. $\theta = \pi/2$ plane.

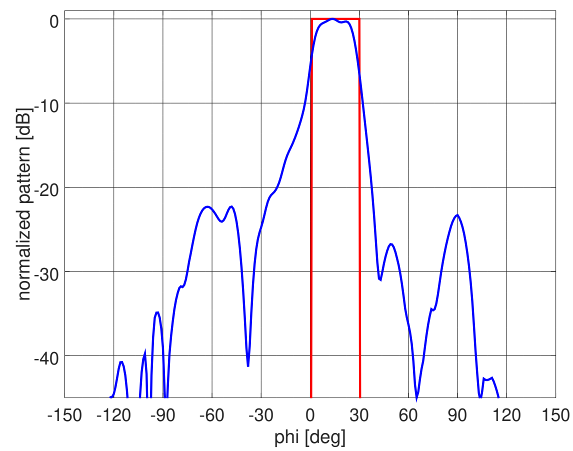

Fig. 3. Pattern from (*blue*) proposed method and (*red*) sector beam specification. $\theta = \pi/2$ plane.

IV. Discussion

Since this method imposes no explicit constraints on the pattern, and does not aim to minimize pattern error in any particular sense, reasonable results are not guaranteed. However it is worth noting that the form of the linear system of equations (Equation 8) makes it simple to add discrete magnitude constraints or derivative constraints (see e.g. [2]) on the pattern. In this case the system will be over-determined, so the additional constraints may not be exactly satisfied. Nevertheless, this could be used as a means to coax the method into delivering a pattern that is closer to that intended.

References

[1] W.L. Stutzman and G.A. Thiele, *Antenna Theory and Design*, 3rd Ed., Wiley, 2013.

[2] M. Er and A. Cantoni, "Derivative Constraints for Broad-Band Element Space Antenna Array Processors," *IEEE Trans. Acoustics, Speech, & Signal Proc.*, Vol. 31, No. 6, Dec. 1983, pp. 1378–93.