Electromagnetic Modeling of Thin Wire with Multilayered Coating in Layered Media

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Abstract—An efficient numerical method is presented to simulate the thin wire structures with multilayered coating in the horizontally layered media. A target application can be underground wireless communication using electromagnetic telemetry (EMT), which consists of a long metal drill string treated as a thin wire and a gap near the drill bit as the source. The numerical scheme uses the electric field integral equation (EFIE) and layered media Green's function (LMGF) to model the electromagnetic interaction of the drill string and layered media. The effect of the multilayered coating (drilling fluid, cement, and casings in borehole) is modeled using the equivalence principle without increasing the number of unknowns. Only the induced current on the thin wire is discretized as the unknowns. Numerical results are presented to demonstrate the efficacy of the proposed method.

I. INTRODUCTION

The thin coating is often used on an antenna or a scatter for protection or insulation purpose. In general, the presence of the coating affects the original radiation characteristics. Thus, accurate modeling of the thin coating is important. In an EMT system, the drill string is coated by multilayered thin materials, including drilling mud, cement and surface casing. To accurately predict the EM signals induced by the drill string, a reliable numerical algorithm is developed.

In the numerical formulation, the current on the drill string is assumed to have only axial distribution which is reasonable at low frequency for EMT system [1]. The long drill string is modeled as a thin wire antenna discretized into 1D segments. Thus, the computational complexity is significantly reduced under this assumption. Then a mixed-potential integral equation (MPIE) form of the electric field integral equation (EFIE) and the method of moments (MoM) are employed to obtain a linear system. The layered media Green's function (LMGF) [2] and thin wire kernel [3] are used to accurately calculate the potentials appearing in the MPIE. To model the surface coating on the drill string, the volumetric equivalence principle (VEP) [4] is applied. The polarization current and charge in the coating are related to the induced current on the drill string using the quasi-static approximation. Therefore, the modeling of the surface coating does not increase the total number of unknowns.

The performance of the proposed numerical method is validated by commercial FEM software COMSOL. The results

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Fig. 1. Schematic of EM telemetry system in layered media.

demonstrate that the proposed method can accurately model a thin wire with multilayered coating in layered media.

II. THEORY AND FORMULATION

The schematic of an electromagnetic telemetry system for underground wireless communication during drilling is depicted in Fig. 1. A metal drilling string surrounded by drilling fluid and casings is inserted in the horizontally layered formation. Each layer has horizontal and vertical conductivity pair, $\sigma_m = (\sigma_{hm}, \sigma_{vm})$ (m = 1, 2, ..., N). A voltage source near the drill bit is emitting the EM waves as the excitation $\mathbf{E}^i(\mathbf{r})$. The problem is formulated as an EFIE,

$$\left[\mathbf{E}^{i}(\mathbf{r}) + \mathbf{E}^{s}(\mathbf{r})\right]_{tan} = 0, \quad \mathbf{r} \in S,$$
(1)

where S denotes the surface of the drill string. We assume the drill string is perfect conducting, i.e., the surface impedance is zero. When the coating is present, the fields due to the coating layers can be assumed to be induced by the volumetric equivalent current and the coating is then replaced by the background layered medium. We consider the volume electric current density \mathbf{J}_{pol} within each coating layer as the equivalent polarization currents. The secondary field $\mathbf{E}^{s}(\mathbf{r})$ can be expressed as the mixed-potential form,

$$\mathbf{E}^{s}(\mathbf{r}) = -j\omega\mathbf{A}_{w}(\mathbf{r}) - \nabla\Phi_{w}(\mathbf{r}) - j\omega\mathbf{A}_{c}(\mathbf{r}) - \nabla\Phi_{c}(\mathbf{r}), \quad (2)$$

where $\mathbf{A}(\mathbf{r})$ denotes the magnetic vector potential and $\Phi(\mathbf{r})$ is the electric scalar potential. The subscript 'w' denotes wire and the 'c' denotes the coating. ω is the angular frequency.

Assuming there are L layers of coatings, the drill string potentials in layered media for electric current sources can be expressed in terms of a mixed-potential form of LMGF [5],

$$\mathbf{A}_{w}(\mathbf{r}) = \mu_{0} \int_{S} \boldsymbol{\mathcal{G}}^{A}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_{\mathbf{s}}(\mathbf{r}') dS', \qquad (3)$$

$$\mathbf{A}_{c}(\mathbf{r}) = \mu_{0} \sum_{i=1}^{L} \int_{V_{i}} \boldsymbol{\mathcal{G}}^{A}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_{pol,i}(\mathbf{r}') dV', \qquad (4)$$

$$\Phi_{w}(\mathbf{r}) = -\frac{1}{j\omega\epsilon_{0}} \bigg[\int_{S} K^{\Phi}(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_{s}(\mathbf{r}') dS' + \int_{S} P_{z}(\mathbf{r}, \mathbf{r}') \hat{\mathbf{z}} \cdot \mathbf{J}_{s}(\mathbf{r}') dS' \bigg],$$
(5)

$$\Phi_{c}(\mathbf{r}) = -\frac{1}{j\omega\epsilon_{0}} \sum_{i=1}^{L} \left[\int_{V_{i}} K^{\Phi}(\mathbf{r},\mathbf{r}') \nabla' \cdot \mathbf{J}_{pol,i}(\mathbf{r}') dV' + \int_{V_{i}} P_{z}(\mathbf{r},\mathbf{r}') \hat{\mathbf{z}} \cdot \mathbf{J}_{pol,i}(\mathbf{r}') dV' \right],$$
(6)

where $\mathbf{J}_{\mathbf{s}}(\mathbf{r}') = \frac{\mathbf{I}(\mathbf{r}')}{2\pi a(\mathbf{r}')}$ is the induced surface current on the drill string, $a(\mathbf{r}')$ is the radius of the drill string, and $\mathbf{I}(\mathbf{r}')$ denotes the total axial current. \mathcal{G}^A is the dyadic Green's function (DGF). K^{Φ} is the corresponding scalar potential kernel, and P_z is the vertical current scalar potential correction factor for layered media [2].

Using the VEP, the polarization current in the *i*-th coating $\mathbf{J}_{pol,i}(\mathbf{r}')$ can be written as [5]

$$\mathbf{J}_{pol,i}(\mathbf{r}') = -\kappa_i (\nabla' \cdot \mathbf{J}_s) \mathbf{\hat{n}},\tag{7}$$

where $\kappa_i = \frac{\epsilon_i - \epsilon_b}{\epsilon_i}$, in which ϵ_i is the effective permittivity of the *i*th coating layer and ϵ_b the effective permittivity of the background formation. $\hat{\mathbf{n}}$ is the unit normal vector pointing from the *i*th layer to (i + 1)-th layer. The thin wire kernel [3] is applied to evaluate the azimuthal integration on the source points at a drill string cross section. At last, we use MoM to get matrix form of the linear system. The current distribution on the drill string is obtained by solving the linear system and then the voltage drop at surface is calculated by numerical integration of the electric field.

III. NUMERICAL RESULTS

We consider a three-layer formation with a half-space filled with air on the top. The horizontal conductivity in each layer is, from top to bottom, 0.5, 0.1, and 0.05 S/m, respectively. The vertical conductivity in each layer is, from top to bottom, 0.1, 0.02, 0.01 S/m, respectively. The interfaces are at z = 400m and z = 700 m. A 1000–m–long vertical drill string has a radius of 12.7 cm. There is a two-layer coating, i.e., L = 2in Eq. (4) and Eq. (6). The first layer is drilling fluid and the second layer is the surface casing. The radius of the two-layer coating is 17.8 cm and 20.3 cm, respectively. The drilling fluid is 1000 m, while the surface casing is 333 m. Furthermore, both water-based mud (WBM) and oil-based mud (OBM) are considered and the corresponding conductivity is 10 S/m and



Fig. 2. Magnitude of electric current distribution on a 1000 m vertical drill string. Left subplot: current distribution for two cases: WBM–casing coating; OBM–casing coating; Central subplot: casing length effect for WBM–casing coating; Right subplot: casing length effect for OBM–casing coating.

0.001 S/m. The conductivity of the surface casing is 10^6 S/m.

Fig. 2 shows the magnitude of the current on a drill string inserted vertically into the formation. In the left subplot, the computed electric current distribution along the drill string agrees well with COMSOL results both for WBM and OBM cases. We notice the current magnitude of the OBM case is much less than the WBM case, which makes perfect sense as the OBM is more resistive. In the central and right subplots, we explore the effect of the surface casing by adding a 800-m casing. For both OBM and WBM scenarios, the surface casing has limited effects on the induced current distribution on the drill string, especially, it is negligible for WBM scenario.

IV. CONCLUSION

A fast and accurate method to simulate thin wire structures with coating is introduced. The coating is modeled using the equivalent principle, which does not increase the number of unknowns. The method can be applied to an arbitrary number of layers of coating on the thin wire.

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REFERENCES

- S. Zeng, D. Li, D. R. Wilton, and J. Chen, "Fast and accurate simulation of electromagnetic telemetry in deviated and horizontal drilling," *Journal* of Petroleum Science and Engineering, vol. 166, pp. 242–248, 2018.
- [2] K. A. Michalski and D. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media. i. theory," *IEEE Trans. Antennas Propag.*, vol. 38, no. 3, pp. 335–344, Mar. 1990.
- [3] D. R. Wilton and N. J. Champagne, "Evaluation and integration of the thin wire kernel," *IEEE Trans. Antennas Propag.*, vol. 54, no. 4, pp. 1200–1206, Apr. 2006.
- [4] S. Zeng, D. R. Wilton, and J. Chen, "Multiscale modeling of electromagnetic telemetry in layered transverse isotropic formation," *IEEE Journal* on Multiscale and Multiphysics Computational Techniques, vol. 3, pp. 266–276, 2018.
- [5] S. Tao, Z. Fan, W. Liu, and R. Chen, "Electromagnetic scattering analysis of a conductor coated by multilayer thin materials," *IEEE Antennas and Wireless Propagation Letters*, vol. 12, pp. 1033–1036, 2013.