Geometric Intuition and Methods for Measuring Closure Phase for Robust Feature Recognition in Interferometric Imaging

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In interferometric imaging applications, closure phase refers to the phase of the product of spatial coherences obtained around a closed loop (polygon with $N \geq 3$ sides) of interferometer elements. Its property of invariance to image translation as well as to phase corruption due to the propagation and instrumental effects that are factorizable into individual element-based terms, and element-based direction-independent phase calibration and errors therein, has been well-known for several decades. The property that this can be measured robustly even using raw and uncalibrated interferometric data makes it a true measurable physical property of the object being imaged, particularly of the degree of symmetry in the intensity distribution in the image. Therefore, it has been a valuable tool in challenging interferometric applications that otherwise require high-accuracy phase calibration. Interesting applications include the EHT imaging of the supermassive black hole event horizon at the center of M87 using very long baseline interferometry, an independent approach to statistical detection of redshifted 21 cm power spectrum of neutral Hydrogen during the epoch of reionization, and optical imaging of stellar surfaces.

Until now, the understanding of interferometric closure phase has been limited to a mathematical description that gets applied primarily in the aperture-plane (the Fourier domain of the image-plane), wherein the corrupting phase terms of the complex gains from individual array elements cancel out, giving a calibration-independent true physical property of the image. However, a geometrical intuition for this valuable physical quantity has been lacking. We provide the foundations for such an insight. We present two geometric methods to measure the closure phase directly from images (without requiring a Fourier- or aperture-plane view) with a simple 3-element array and the image-plane interferometric responses ("fringes"):

1. the closure phase is directly measurable from the positional offset of the null phase curve (NPC) of one fringe from the intersection of the other two fringe NPCs, and
2. the squared closure phase is proportional to the product of areas enclosed by the triad of detectors and the three fringe NPCs in the aperture- and image-planes, respectively.

We validate this new formalism using radio interferometric observations of the bright radio quasar 3C 286 using the Jansky Very Large Array (JVLA) radio telescope by verifying the direct and geometrical method in the image-plane against the conventional aperture-plane method of determining closure phase.

The geometric understanding provided herein can be potentially valuable to optical interferometry and other interferometric applications. We further generalize these geometric relationships to an N-element interferometer.