Stochastic Analysis of Human Exposure Assessment by Surrogate Model

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***Abstract*—Despite recent advances in the field, the stochastic analysis of EM models is still time-consuming. Thus, the approximation of the EM model by a surrogate model is often the only choice if a sufficient amount of simulations is required. This paper provides a systematic and novel methodology to implement the polynomial chaos expansion surrogate models in stochastic analysis. The necessary steps in the construction of surrogate model based on polynomial chaos expansion are explained. As an example, we estimate the statistics of the specific absorption rate of a smart watch GSM antenna on a human wrist. Results show that the proposed method is accurate and time-saving in the cases when numerous rounds of simulation are required.**

Keywords—GSM antenna; Monte-Carlo method; specific absorption rate; stochastic analysis; surrogate model

I. Introduction

To describe the electromagnetic problems, we typically use deterministic models, such models are assigned to definite values and we find a deterministic solution to the problem. In reality, however, those problems mostly have uncertainties, like indefinite parameters, imprecise experimental results, etc. [1]

The simplest approach to study the model uncertainty is Monte-Carlo analysis. Monte-Carlo analysis yields reasonable results only if the number of samples is quite large, which requires a great computational effort if full-wave simulations are applied. To avoid long simulation time, we consider using surrogate models to replace full-wave simulations in Monte-Carlo analysis. Advances in machine learning research provide mathematical models which are used to generalize and predict the results of highly nonlinear and multivariable problems. This paper provides a systematic methodology to implement one of the most popular models, polynomial chaos expansion (PCE) surrogate model in Monte-Carlo analysis of an electromagnetic design. In general, PCE surrogate model relies on well-established experimental or simulation results decomposed in the form of polynomial interpolation.

The aim of the paper is to consider the variation of the outputs of an EM system induced by the variations of the inputs. We denote full-wave simulation models as $y = M(\boldsymbol{x})$, and surrogate models of PCE as $\hat{y} = \widehat{M}(\boldsymbol{x})$, with the inputs $\boldsymbol{x}$ affected by some possible random variations. It is worth noting that random variables $\boldsymbol{x}$ have to be statistically independent when applied in PCE.

In this paper, Section 2 explains the construction of PCE surrogate model. Section 3 discusses a benchmark of human exposure stochastic analysis, in which a surrogate model estimates the specific absorption rate (SAR) of a GSM antenna in smart watch placed on a human wrist. Monte-Carlo analysis is performed on the surrogate model to characterize the SAR statistics with respect to human morphology variation.

II. Surrogate Model of PCE

A. Data Sampling

To build a surrogate model, full-wave simulations are performed on a point set $\{\boldsymbol{x}_e\}$ to get the experiments' data $y(\boldsymbol{x}_e)$. The sampling set has two constraints: 1. The sampling is as uniformly distributed as possible over the entire input parameter space to capture possible nonlinearities of the simulated phenomena; 2. The number of simulations is large enough to estimate all the coefficients of the surrogate model, but limited to reduce the cost of the number of simulations. This paper implements Latin Hypercube Sampling (LHS), which generates a sample set of parameter values from a multidimensional statistical distribution while taking care of a uniform filling of space. As an illustration of the sampling, a two-dimensional space is divided into square grids, based on equally probable intervals. LHS contains the sample positions if there is only one sample in each row and each column.

B. Model Selection

Generally, surrogate models of PCE $\hat{y} = \widehat{M}(\boldsymbol{x})$ with degree of K can be expressed as [2]:

$$\hat{y} = \sum_{k=0}^{K-1} \beta_k \psi_k(\boldsymbol{x}) \tag{1}$$

Here $\{\psi_k(\boldsymbol{x})\}$ is a set of orthogonal multivariable polynomials defined by $\psi_k(\boldsymbol{x}) = \prod_{i=1}^{N} \pi_{k_i}^{i}(x_i)$, and the order of the multivariable polynomials is the sum of the order of each variable, $\sum_{k_i=1}^{N} k_i = k$. PCE is formulated with uniform random variables with basic function $\pi_{k_i}(x)$ as Legendre polynomials, which has the following properties:

$$P_{-1}(x) = P_0(x) = 1 \tag{2}$$

$$(n+1)P_{n+1}(x) = (2n+1)xP(x) - nP_{n-1}(x) \quad n \in N \tag{3}$$

C. Coefficient Calculation

The coefficients $\{\beta\}$ in PCE are calculated by fitting the model response $\hat{\boldsymbol{y}}_{\boldsymbol{e}}$ with the full-wave simulation results $\boldsymbol{y}_{\boldsymbol{e}}$. The calculation by least-mean-square regression aims at computing PCE coefficients that minimize the mean-square error of approximation of the model response. The least-mean-square regression calculates the coefficients as $\beta = (\Psi^T\Psi)^{-1}\Psi^T\boldsymbol{y}$.

One drawback of the least-mean-square regression is that every term in expansion has a non-zero coefficient although some polynomials may have little importance in the expansion. The least absolute shrinkage and selection operator (*lasso*) regression adds a penalty equal to the absolute value of coefficients. The goal of the algorithm is to minimize

$$\sum_{i=1}^{n}\left(y_i - \sum_{k}\psi_{ij}\beta_k\right)^2 + \lambda\sum_{k=1}^{K-1}|\beta_k| \qquad (4)$$

This type of regulation can set some coefficients as zero and eliminate the polynomials from the model, resulting in sparse models with fewer polynomials.

D. Model Verification

The use of alternative models requests a method to validate these surrogate models. This paper uses the leave-one-out cross-validation (LOOCV), which is often used in computer experiments. Assume you have n experiments $y_i = M(x_i)$, you can use $n-1$ experiments to build a model and one experiment to test it. If we consider the n simulations, $\boldsymbol{y} = \{y_1, y_2, \dots, y_p, \dots, y_n\}$ that have been performed and if one notes $\widehat{M}_{-i}$ the model based on $n-1$ simulations, $\{y_1, y_2, \dots, y_n\} - \{y_j\}$, we can estimate the mean square error of the model using

$$err_{loo} = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{M}_{-i}(x_i) - M(x_i)\right)^2 \qquad (5)$$

III. Benchmark of SAR Characterizations

To verify the compliance of wireless systems with safety limits when used close to human body, it is of interest to characterize SAR. Since the human population is highly diverse and it is impossible to generalize any result obtained on one model, the object of this benchmark is to analyze the statistics of SAR in terms of morphology variation. The approaches that use full-wave simulation are not suitable for the Monte-Carlo method. To overcome this limitation, a surrogate model with sparse PCE has been used.

This benchmark characterizes the SAR statistics based on such a configuration that the radiation of a GSM antenna in a smart watch is absorbed by a human wrist. IEEE Std. C95.1 standard sets the limit of the exposure to wrists as 4W/kg 10-g volume averaged SAR (SAR10g). One feasible way to embed a GSM antenna in a watch is to use the watch strap as a dipole [3]. The antenna operates on the GSM 900 band and the equivalent radiated power of the GSM antenna is 0.25W RMS. Fig. 1 shows the biological model representing the morphology of most human beings' wrists. There are totally seven geometric parameters that determine the sizes and locations of bones and vessels in a wrist. We provide three scenarios of GSM dipole antenna placements as shown in Fig. 2.

A surrogate model is generated and a Monte-Carlo test with 1000 evaluations is operated for each scenario. Fig.3 shows the histograms of the SAR10g occurrence. We can see that if the antenna is placed just on top of the skin, the SAR10g is very likely (96.2%) to exceed the 4 W/kg limit. If we place the GSM dipole on top of a 1mm thick watch strap, we still get a large chance (59.9%) that the SAR10g does not satisfy the 4W/kg criterion. If the GSM dipole is placed on top of a normal watch strap with 2.5mm thickness, the SAR10g's of all of the samples are less than the criterion.

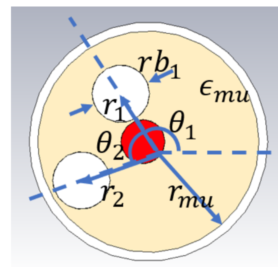
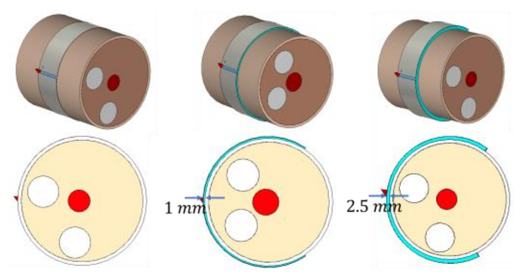
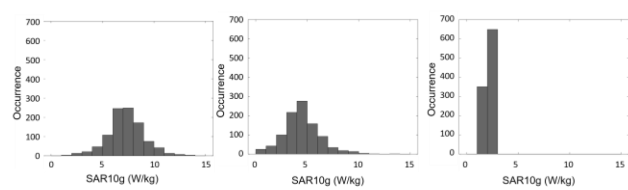


Fig. 1. Human wrist model. Its morphology is controled by seven parameters.



(a) (b) (c)

Fig. 2. Three scenarios of a GSM dipole antenna on a smart watch. (a) The dipole is placed underneath a dielectric watch strap and directly touches skin, (b) the dipole is placed on top of a dielectric watch strap, the strap thickness is 1mm, (c) the dipole is placed on top of a dielectric watch strap, the strap thickness is 2.5mm.



(a) (b) (c)

Fig. 3. Histograms of Monte-Carlo analysis with 1000 surrogate model evaluations in the cases of (a) the antenna underneath a strap, (b) antenna placed on top of a 1mm strap, (c) antenna placed on top of a 2.5mm strap.

References

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