Irregularity Parameter Estimation for Interpretation of Scintillation Doppler and Intensity Spectra

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***Abstract*—We investigate the Irregularity Parameter Estimation (IPE) technique for characterizing ionospheric irregularities using satellite beacon observations of amplitude and phase scintillation. The approach is to fit either the Doppler spectrum or the intensity spectrum of scintillation observations with theoretical fitting functions derived for the case of propagation through a thin phase changing screen. Fitting the Doppler spectrum yields estimates for the phase spectral strength, spectral index, and effective scan velocity. Fitting the intensity spectrum provides estimates for these parameters, and additionally the Fresnel scale. We find that fitting the intensity spectrum tends to provide more robust and accurate results than fitting the Doppler spectrum. Nevertheless, fitting the Doppler spectrum is simpler and generally produces acceptable estimates of phase spectral strength and spectral index.**

I. INTRODUCTION

Irregularity Parameter Estimation (IPE) is an inverse radio propagation technique in which the statistical characteristics of ionospheric irregularities are extracted from the time-series of scintillations they produce [1]-[3]. These characteristics may be expressed in terms of phase spectral strength and spectral index, and the effective scan velocity which maps spatial frequencies in the screen to temporal frequencies. This effective scan velocity is related to the irregularity drift, satellite motion and magnetic field geometry. The IPE technique has been applied to time series of intensity scintillations [1]-[2] and also in-situ density observations [3]. A variation of the IPE technique has been used to estimate the mean distance to irregularities responsible for producing scintillations along radio-occultation (RO) ray-paths [4]. In this paper, we apply the IPE technique to Doppler spectra and compare the estimated phase screen parameters with those estimated via application of IPE to intensity spectra.

II. DISCUSSION

Carrano and Rino [3] described a 2D model for the spectral density function (SDF) of intensity fluctuations in the receiver plane following propagation of a plane wave through a thin phase-changing screen:

$$\Phi_I(q) = \int_{-\infty}^{\infty} \exp\left\{-g\left(r, q\rho_F^2\right)\right\} \exp\left(-iqr\right) dr , \qquad (1)$$

where q is spatial wavenumber, r is spatial separation, ρ_F is the Fresnel scale and g is the so-called structure interaction function:

$$\begin{aligned} g(r_1, r_2) &= D_{\delta\phi}(r_1) + D_{\delta\phi}(r_2) \\ &\quad - \tfrac{1}{2} D_{\delta\phi}(r_1 + r_2) - \tfrac{1}{2} D_{\delta\phi}(|r_1 - r_2|) \end{aligned} \qquad (2)$$

In the above, $D_{\delta\varphi}(r)$ is the structure function of phase in the screen. A similar expression may be derived for the SDF of Doppler variations in the receiver plane:

$$\Phi_D(q) = \int_{-\infty}^{\infty} \exp\left\{-\tfrac{1}{2} D_{\delta\phi}(r)\right\} \exp\left(-iqr\right) dr . \qquad (3)$$

The Doppler spectrum is the Fourier transform of the mutual coherence function $R_u(r) = \langle u_1(x) u_2^*(x+r) \rangle$ where u is the complex amplitude in the receiver plane and '*' denotes complex conjugate.

For simplicity of presentation, we consider a phase screen characterized by an unmodified power law spectrum. In this case, the structure function is a function of phase spectral strength C_p' and phase spectral index p:

$$D_{\delta\varphi}(r; C_p', p) = C_p' \frac{2^{2-p} \Gamma\left[(3-p)/2\right]}{\sqrt{\pi}(p-1)\Gamma\left[p/2\right]} |r|^{p-1}, \ 1 < p < 3, \qquad (4)$$

where Γ is the gamma function. We assume temporal frequencies, f, are related to spatial wavenumbers through $f = V_{eff}(q/2\pi)$, where V_{eff} is the effective scan velocity [5]. Substituting this and the phase screen model (4) into equations (1)-(3) yields a model for the temporal SDF of Doppler fluctuations

$$D(f; C_p', p, V_{eff}) = 2\int_0^{\infty} \exp\left\{-\tfrac{1}{2} D_{\delta\phi}(r; C_p', p)\right\} \cos\left(\frac{2\pi f}{V_{eff}} r\right) dr, \qquad (5)$$

and a model for the temporal SDF of intensity fluctuations

$$I(f; C_p', p, \rho_F, V_{eff}) = 2\int_0^{\infty} \exp\left\{-g\left(r, \frac{2\pi f}{V_{eff}} \rho_F^2; C_p', p\right)\right\} \cos\left(\frac{2\pi f}{V_{eff}} r\right) dr. \qquad (6)$$

These models may be used to fit observed Doppler and intensity scintillation spectra for any strength of scatter. In general, they must be evaluated via numerical quadrature. As in [2] we use the maximum likelihood method to fit the spectra. It is readily shown that maximizing the likelihood of measuring scintillation with Doppler SDF $D^{measured}(f)$ and intensity SDF $I^{measured}(f)$, given the models (5) and (6), is equivalent to minimizing the following summations over frequency as a function of their respective arguments:

$$S_D(C'_p, p, V_{eff}) = 2\sum_{i=1}^{N-1}\left\{\frac{D^{measured}(f_i)}{D(f_i)} + \ln[D(f_i)]\right\}, \quad (7)$$

$$S_I(C'_p, p, \rho_F, V_{eff}) = 2\sum_{i=1}^{N-1}\left\{\frac{I^{measured}(f_i)}{I(f_i)} + \ln[I(f_i)]\right\}. \quad (8)$$

In the above, the f_i's are the FFT frequencies and N is the number of samples. Fitting the Doppler SDF yields estimates for C_p', p, and V_{eff}, while fitting the intensity SDF yields estimates for these parameters and also the Fresnel scale ρ_F.

To quantify the accuracy of the IPE fitting, rather than using real scintillation measurements, we use simulated scintillation produced via forward phase screen simulation with known screen characteristics C_p'=0.001, p=2.5, V_{eff}=50 m/s, and ρ_F=100m. We generated realizations of complex amplitude in the receiver plane, computed Doppler and intensity, and then their spectra via fast Fouier transform (FFT). Figs 1-2 show the variation of S_D and S_I near their respective minima, shown with red dots. The logarithm of the relative variation in (7) and (8) is shown for clarity. The 'truth' values for the parameters are indicated with red dashed lines. Note that S_D has a well-defined minium in terms of C_p' and p but in terms of V_{eff} we see a long trough containing many local minima. This suggests fitting the Doppler spectrum may not provide robust estimates for V_{eff}, in general. From Fig 2 we see that S_I has a well-defined minimum in terms of all four parameters (including V_{eff}). Moreover, the values of the parameters at the minimum of S_I are closer to their true values. While we have shown results for a single realization only, we did note this to be a general trend.

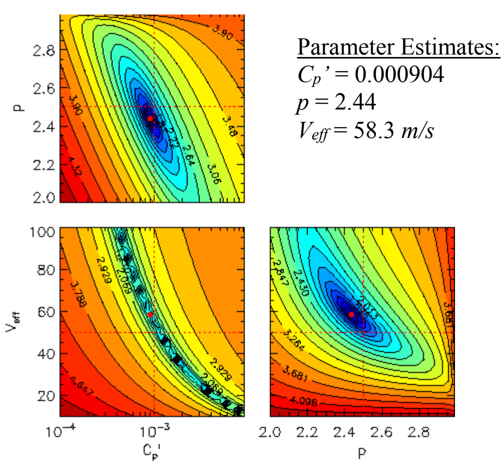

Fig. 1. Variation of S_D (log relative units) as a function of phase screen parameters C_p', p, and V_{eff}.

Fig 3 shows the spectral fits obtained by minimizing S_D and S_I, respectively. Both the Doppler and intensity SDFs exhibit power law behavior in the high frequency range. The intensity SDF exhibits Fresnel filtering while the Doppler SDF does not.

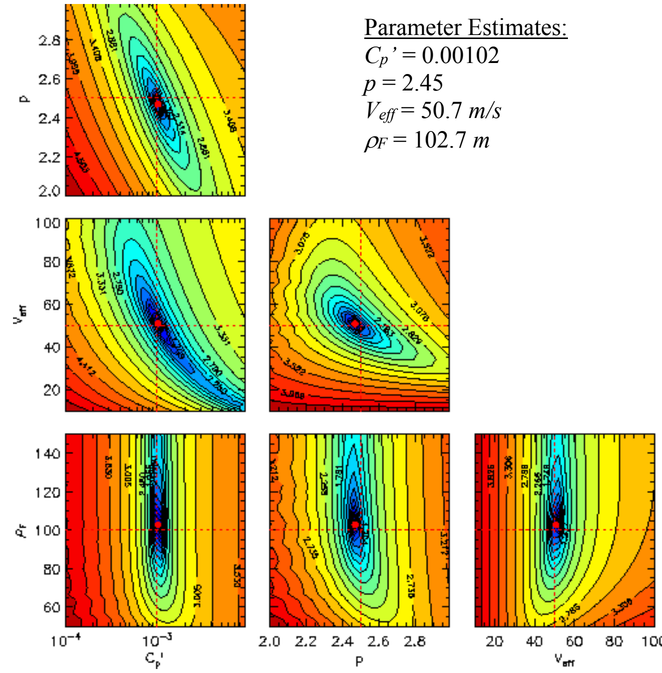

Fig. 2. Variation of S_I (log relative units) as a function of phase screen parameters C_p', p, V_{eff} and Fresnel scale ρ_F.

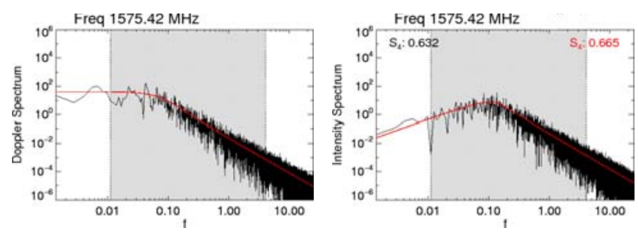

Fig. 3. IPE fits (red) for the simulated (black) Doppler spectrum (left) and simulated intensity spectrum (right).

In summary, we find that fitting the intensity spectrum provides more accurate results than fitting the Doppler spectrum. Nevertheless, fitting the Doppler spectrum produces adequate estimates of phase spectral strength and spectral index. When we present this paper, we will show results of IPE fitting using the two-component model described in [1] and using scintillation observations from the Global Positioning System (GPS).

References

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