Propagation in Highly Anisotropic Random Media

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***Abstract*—Propagation in randomly irregular media is governed by the parabolic wave equation, which supports both simulations and complex signal-moment characterization. Until very recently, both simulations and theory modeled the anisotropic medium with a three-dimensional homogeneous structure with ellipsoidal contours of constant correlation. This paper investigate propagation through a distribution of physical *striations*.**

I. INTRODUCTION

In a recent paper [1] a configuration-space ionospheric structure model was introduced. Configuration-space structure realization are summations of randomly located *striations*. A striation is an elemental structure contribution associated with a single field line. The ionization is defined by a product of radial and axial profile functions scaled by a size parameter and peak electron density. By choosing a power-law size and fractional-strength variations, the two-dimensional spectral density function (SDF) in slice planes penetrated by the striations has a complementary power-law SDF. With negligible field-line variation over the data-space, the stochastic structure is manifest in two-dimensional slice planes.

The configuration-space model was designed to generate more representative ionospheric structure realizations for propagation simulations. This presentation summarizes preliminary results. Equivalent-phase-screen models, which replace propagation through extended structure with a path-integrated structure assigned to a single phase screen, are evaluated. Two-dimensional propagation models that confined the propagation computation to the plane containing the propagation vector and the direction of translation are also investigated. Two-dimensional model are appealing because they admit tractable analytic intensity SDF calculations [2], and they can be applied directly to data interpretation [3].

II. PHASE-SCREEN EQUIVALENCE

Split-step integration of the parabolic wave equation was employed, with the refractive index perturbations derived from the configuration-space model. A reference coordinate system with the x axis along the propagation direction is used. The rectangular data-space is populated with striations with arbitrary orientation, whereby propagation at any angle relative to the striation axes can be simulated. The current version of the model assumes parallel field lines within the data space.

Realizations were generated for propagation at 90°, 30°, and 0° with respect to the magnetic field direction. Striation locations were randomized in the defining central intercept planes. For the propagation simulations the yz plane dimensions were 200×50 km with 4096×4096 samples. The extent of the disturbed region was 30 km. The size of the data volume was chosen to be representative. Figure 1 shows the intensity fields in the observation plane at $x = 150$ km from the center of the disturbed region.

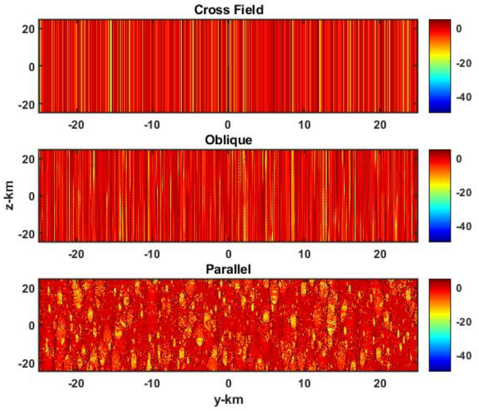

Fig. 1. Intensity field for cross-field, oblique, and field-aligned propagation.

The left frame of Figure 2 shows the development of the intensity as measured by the $S4$ scintillation index. The red circles show the full integration development through the disturbed region followed by free space propagation to the observation plane. The blue circles show the development from a an equivalent phase screen placed at the center of the structured region. The equivalent-phase-screen structure develops more quickly, effectively *catching up* with the full diffraction result at the transition to free space.

The center and right frames show the intensity and phase from cuts through the diffraction patterns in the observation planes at $z = 0$. The center frame is from the full diffraction simulation. The right frame is from the phase screen simulation. The equivalence persists to near-field-aligned propagation where the diffraction pattern of the striations seen in the lower frame of Figure 1 dominates the field structure.

III. DIAGNOSTIC MEASUREMENTS

Diagnostic measurements are effectively one-dimensional scans through the two-dimensional diffraction pattern in an observation plane. In the absence of prior information, the two-dimensional structure characteristics cannot be determined.

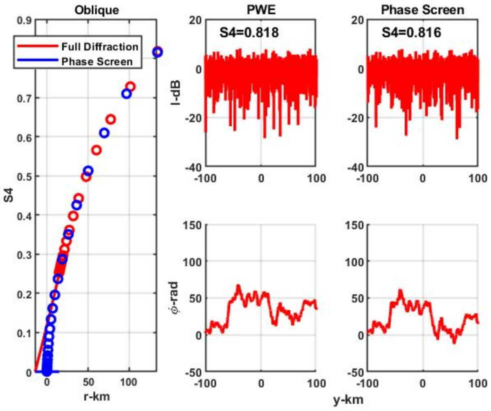

Fig. 2. Cross-field summary: Left frame PWE (red) and phase-screen (blue) S4. Solid lines are full diffraction. Circles are free space. Middle and right frames summarize intensity and phase at $z = 0$.

However, if the diffraction pattern remains highly anisotropic and the propagation angles relative to the magnetic field are known, it is possible to estimate the structure defining parameters and the central location of the structure. Figure 3 shows a path-integrated phase pattern, which maps onto the diffraction pattern in the measurement plane. For the same frequency and propagation distance, the scintillation intensity development is identical to that summarized in Figure 2.

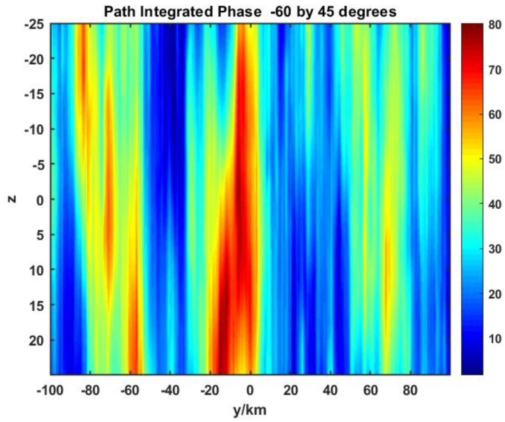

Fig. 3. Oblique Path-Integrated Phase

The upper frame of Figure 4 summarizes the scintillation structure measured along the $z = 0$ scan. The phase scintillation SDF (blue) is overlaid on the corresponding SDF of the one-dimensional phase slice (cyan). The intensity SDF is plotted in (red). The lower frame of Figure 4 shows the intensity spectrum obtained by initiating a two-dimensional propagation calculation with the one-dimensional phase. The phase SDFs are identical to those shown in the upper frame. The intensity SDF was obtained by varying the propagation distance until the S4 and intensity SDF was in qualitative agreement with the intensity SDF shown in the upper frame. For comparison the intensity SDF from the one-dimensional scan of the three-dimensional realization is overlaid in green.

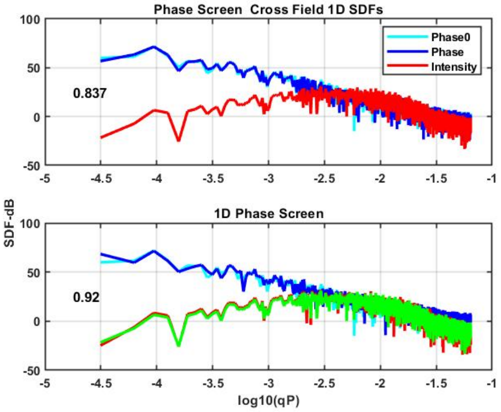

Fig. 4. 3D-2D Equivalence

IV. CONCLUSIONS

The preliminary analysis of the configuration space propagation simulations show that for a uniform structure distribution along the propagation path replacing the PWE integration with a single path-integrated structure produces statistically equivalent results up to near-field-aligned propagation. Moreover, preliminary results evaluating the statistical equivalence of two-dimensional propagation initiated by a one-dimensional phase screen are encouraging. Results of ongoing work will be presented.

REFERENCES

[1] C. Rino, C. Carrano, K. Groves, and T. Yokoyama, “A configuration-space model for intermediate-scale ionospheric structure,” *Radio Science*, 2018, accepted for publication.

[2] C. S. Carrano and C. L. Rino, “A theory of scintillation for two-component power law irregularity spectra: Overview and numerical results,” *Radio Sci.*, vol. 51, no. doi:10.1002/2015RS005903, p. 789813, 2016.

[3] C. Rino, B. Breitsch, Y. Morton, Y. Jaio, D. Xu, and C. Carrano, “A compact multi-frequency gnss scintillation model,” *J. Ints. of Navigation*, 2018, accepted for publication.