A Modal Analysis of Scattering of Objects in an Inhomogeneous Waveguide

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Accurate results for scattering from an obstacle in a waveguide are based on numerical schemes such as finite element method or method of moments. However, there exist several approximate methods which offer more physical insight, albeit in a restricted domain. In our problem we have an inhomogeneous waveguide in which a multitude of objects are embedded. A distant source excites these objects and our interest is to understand the energy distribution in the modes well away from the domain containing the objects. Although multiple scattering has been extensively studied for the case of objects in a homogeneous open domain, not much has been done on problems when objects are embedded in an inhomogeneous waveguide.

Direct numerical approach to this problem is not only labor-intensive, but it is also not conducive to physical understanding. Suppose we know the scattering characteristics of each object in isolation when placed in an unbounded homogeneous background. For example, we assume that we know its transition matrix or scattering matrix. The obvious representation for such quantities is terms of spherical wave functions. However, the source signal and scattered signals are naturally represented in terms of cylindrical mode functions. Hence a major task of this problem is to transform the incident cylindrical wavefunctions to spherical wave functions and to transform the scattered spherical wave functions to cylindrical wave functions. We thus obtain the desired representation of scattering of an object in waveguide environment. Note that in this representation, part of multiple scattering between the object and waveguide is ignored. However, with this representation we can readily proceed to carry out multiple scattering analysis similar to the situation of objects in an unbounded domain.

The above procedure is good for the case when the waveguide is homogeneous. However, when it is inhomogeneous, it is not clear how to employ the transition matrix representation of the object. For such problems, some researchers have used the artifact of enclosing the object in a shell within which the medium is homogeneous. It is claimed that this approximation is good when the waveguide is weakly inhomogeneous. Our procedure is to represent the mode functions of the inhomogeneous waveguide in terms of those of a homogeneous waveguide. In this approach the transition matrix representation is accurate and there is no restriction on the degree of inhomogeneity of the waveguide. We illustrate this with the help of some numerical examples.