A method of a tangent cylinder in the theory of wave scattering by concave surfaces.

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In 1952 L.M Brekhovskikh put forward an approximate solution of the problem of waves scattered from rough surfaces which he termed the tangent-plane approximation (TPA). In this approximation at each point of a smooth surface the latter is approximated by a tangent plane at the point. Since the field reflected from a plane can be calculated analytically, both the total field and its normal derivative at each point became known due to the Fresnel-type formulas, and the Helmholtz integral provides scattered field at any point in the space. Along with the method of small perturbations, the TPA became one of two cornerstones of the theory of scattering of waves from rough surfaces and is widely used.

It is clear that the TPA represents a short-wave (high-frequency) asymptote of the exact solution of the scattering problem. It is valid only for boundaries with sufficiently small curvature and sufficiently large incidence angles. Thus, at the surface of a compact body there always exist points where the incident wave vector is tangent to the surface and the criterion of applicability is violated. Related to this is a treatment by the TPA of the field in the shadow zone, which is usually set to zero.

A natural way to improve the TPA is to increase the accuracy of the approximation of the surface. In a 2D case this leads to the “tangent-cylinder approximation” (TCA), which works just as the TPA with the only difference being that the smooth surface of an object is approximated at each point by a tangent cylinder with a curvature coincident with the local curvature of the boundary at this point. The total field and its normal derivative at a surface of a cylinder can be calculated by a relatively simple explicit formula which represents a sum over cylindrical harmonics. Since the cylindrical functions and their derivatives for higher indices can be calculated using simple recursions, the corresponding calculation is in fact not time consuming. In this talk the TCA is applied and verified numerically in the case of an elliptical surface with an impedance boundary condition. Order-of-magnitude improvement of the accuracy of the solution obtained by TCA as compared with TPA is observed.