Analogy Between Elastodynamic Displacement and Electromagnetic Vector Potentials

John W. Neese*[(1)], David R. Jackson[(2)], and Leon A. Thomsen[(1)]

(1) Department of Earth and Atmospheric Sciences
University of Houston, Houston, TX 77204-5007

(2) Department of Electrical and Computer Engineering
University of Houston, Houston, TX 77204-4005

This presentation demonstrates a modeling analogy, for linear homogenous isotropic media, between the displacement vector **u** in elastodynamics (ES) and the vector potentials **A** and **F** in electromagnetics (EM). The development shows that when using the usual Lorenz gauge in EM, the resulting differential equations for **A** and **F** are special cases of the differential equation for **u**, where the medium in the ES problem is constrained by a relation between the ES Lamé parameters λ and μ, namely $\lambda + \mu = 0$. This means that any EM problem can be modeled as an ES problem, but not every ES Problem can be modeled as an EM problem (only those that satisfy the above constraint). However, it is shown that by choosing a generalized Lorenz gauge that allows for an extra degree of freedom, the differential equations for **A** and **F** become completely analogous to the differential equation for **u**, and hence any ES problem is equivalent to an EM problem and vice versa. With this new choice of gauge, the vector potentials **A** and **F** can be expressed as the sum of irrotational and solenoidal parts that travel at different velocities, just as the *P*-wave and *S*-wave components of **u** do in ES. The electric and magnetic fields (the physical fields) do not contain the irrotational part, since the curl of the vector potential is used to obtain these fields.

The analogy leads to an analogy between ES and EM plane waves. The simplest ES case is chosen for illustration: elastic horizontal (y-polarized) sheer (SH) waves traveling in the x-z plane. In this case, the SH waves can be modeled as transverse electric TE_y EM plane waves (also TM_z in the x-z plane) using an electric vector potential component that has only a y component, F_y. The SH mechanical impedance, being the ratio of force over velocity, is analogous to TE_y electrical impedance, being the ratio of E_z over H_y. The impedance analogy follows directly from the ES to EM analogy mentioned above, after a simple calculation to get the fields E_z and H_y from F_y.

For an SH plane wave in horizontally-layered media, we can use the analogy with plane TE_y waves to model an SH wave in a given layer by a transmission line that corresponds to that layer. The SH wave in a multilayer structure can then be modeled using a Transverse Equivalent Network (TEN). All of transmission line theory then becomes available to help us model SH waves in multilayer structures. One use of the TEN is to derive a Transverse Resonance Equation (TRE) that gives the transcendental equation for the unknown wavenumber of a seismic Love wave.