The Antenna Equation: A Description of Antennas Inspired by Scattering Parameters

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Antenna performance is often described in terms of gain or realized gain. This is adequate for many purposes, however, it lacks phase information that is needed, for example, in phased arrays. Such applications commonly use effective length, which is the ratio the received open circuit voltage to the incident electric field (IEEE, *IEEE Standard Definitions of Terms for Antennas*, IEEE Std 145™–2013, Institute for Electrical and Electronics Engineering, New York, December 2013).

However, effective length has two problems. First, it is not completely general for all cases of interest. It does not work well with waveguide feeds, since it is difficult to measure an open-circuit voltage across a waveguide. The normal fix for this is to characterize the antenna after adding a waveguide-to-coax adapter. However, this is an incomplete solution, because one might add waveguide circuitry to the front end of the antenna. If one wishes to isolate antenna performance from, for example, a waveguide filter, that is impossible with a formulation using effective length.

A second problem with effective length is that it produces equations that are unnecessarily complicated. This complexity affects little in the frequency domain, however, the equations become unwieldy in the time domain.

To achieve maximum simplicity and generality, we introduce the antenna equation (E.G. Farr, "Characterizing Antennas in the Time and Frequency Domains," *IEEE Antennas and Propagation Magazine*, February 2018, pp. 106-110) and (E. G. Farr, "A Power Wave Theory of Antennas," *Forum for Electromagnetic Methods and Application Technologies* (online), Vol. 7, 2015, www.e-fermat.org). The antenna equation describes antenna performance in a manner that is both complete and elegant. It works in both the time and frequency domains, and in both transmission and reception. It also adds a meaningful phase to antenna gain and radar cross section. Finally, it works well with waveguide feeds.

The antenna equation has at its core a Generalized Antenna Scattering Matrix (GASM), which is a direct analog of scattering parameters (S-parameters) in microwave theory. As with S-parameters, the GASM works well with waveguide feeds. The equations describe antenna performance as simply as possible. This is crucial when the equations are converted to the time domain, because it eliminates an extra convolution that is associated with a matching circuit.

The antenna equation works well with signal flow graphs. Thus, Mason's rules apply to circuits containing antennas. One can easily calculate the antenna response in a variety of situations, including sources and loads with arbitrary impedance, and antenna scattering under a condition of arbitrary load. One can also add matching circuits to the antenna port and calculate their effect. Several examples will be provided.