Adjoint-based A Posteriori Error Estimation and its Applications in CEM: DHO FEM Techniques and 3D Scattering Problems

Jake Harmon*[(1)], Cam Key[(1)], Blake Troksa[(1)], Troy Butler[(2)], Donald Estep[(3)], and Branislav M. Notaroš[(1)]
(1) Colorado State University, Department of Electrical & Computer Engineering, Fort Collins, CO
(2) University of Colorado, Department of Mathematical and Statistical Sciences, Denver, CO
(3) Colorado State University, Department of Statistics, Fort Collins, CO
j.harmon@colostate.edu, camkey@rams.colostate.edu, blake.troksa@gmail.com, troy.butler@ucdenver.edu, donald.estep@colostate.edu, notaros@colostate.edu

This paper addresses the development and applications of a posteriori uncertainty quantification and error estimation for three-dimensional (3D) electromagnetic scattering problems simulated by frequency-domain double (geometrical and field-approximation) higher order (DHO) finite element method (FEM). Our approach relies on the dual-weighted residual (DWR) for the computation of error estimates, refinement of meshes, and efficiency improvement for problems requiring multiple simulations on the same mesh. Although common in many fields of computational science and engineering including computational fluid dynamics and structural mechanics, such adjoint-based, a posteriori uncertainty quantification and error estimation techniques have seen little practical application in frequency-domain CEM, let alone DHO frequency-domain FEM.

We first discuss the DWR and its effective implementation using DHO FEM methodologies for 3D scattering problems. Giving a general statement of the scattering problem in terms of the 3D double-curl wave equation, we present a brief derivation of the adjoint problem and associated measurement characteristic for an example quantity of interest (QoI), emphasizing the general derivation technique and its extension to other implementations.

We then introduce an example problem, plane wave scattering from a lossy dielectric sphere in free space, and its formulation and discretization in DHO FEM for demonstration of the DWR a posteriori error estimation technique and its applications. This simple example is chosen due to its ubiquity as a test case in CEM and the availability for an analytical expansion of the solution to which DWR error estimates can be compared. We present error estimates computed by the DWR for this problem and discuss their behavior.

We further demonstrate the usefulness of the DWR technique for efficient discretization refinement in the DHO FEM technique, giving convergence results and evaluating error and performance changes for discretization refinement and coarsening. We then extend this demonstration to show the saturation of the discretization for a multi-solve problem on the lossy dielectric sphere mesh, showing the net performance increase with respect to the conventional approach.