Manipulation of Fresnel Coefficients using Crossanisotropic Metasurface Coating

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***Abstract*—A *crossanisotropic* metasurface placed at the interface between two media is studied. Crossanisotropic metasurfaces are a subset of bianisotropic metasurfaces, where only the cross-coupling (electric-to-magnetic and magnetic-to-electric) susceptibility components are non-zero. It is shown that such metasurface coating can be engineered to achieve matching at arbitrary angles of incidence and, more generally, to manipulate the conventional Fresnel (reflection and transmission) coefficient functions.**

I. Introduction

Metasurfaces possess seemingly limitless capabilities to transform electromagnetic waves, given their many degrees of freedom and the recent development of efficient tools to control them [1]. They have been demonstrated in numerous applications, such as for instance generalized refraction and reflection [2]–[5], and many others.

Very recently, some researches have dedicated efforts to applying metasurfaces for *matching*, whereby the metasurface is placed – as a sort of *coating*– at the interface between the media to match [6], [7]. However, the matching metasurface coatings reported in these works were restricted to normal incidence, and did not exploit the full potential of metasurface coating.

Here, we extend metasurface matching to *arbitrary angles of incidence* using *crossanisotropic* metasurfaces (metasurfaces having only cross-coupling susceptibilities), and we show that such metasurfaces more generally allow to *manipulate the Fresnel coefficient functions*.

II. Crossanisotropic Susceptibilities

Metasurfaces are best modeled by Generalized Sheet Transition Conditions (GSTCs). The GSTCs for a general bianisotropic metasurface read [8]

$$\hat{z}\times\Delta\mathbf{H} = j\omega\epsilon\overline{\overline{\chi}}_{\text{ee}}\mathbf{E}_{\text{av}} + jk\overline{\overline{\chi}}_{\text{em}}\mathbf{H}_{\text{av}}, \tag{1a}$$

$$\Delta\mathbf{E}\times\hat{z} = jk\overline{\overline{\chi}}_{\text{me}}\mathbf{E}_{\text{av}} + j\omega\mu\overline{\overline{\chi}}_{\text{mm}}\mathbf{H}_{\text{av}}, \tag{1b}$$

where the Δ symbol and the 'av' subscript represent the differences and averages of the tangential fields at both sides of the metasurface, and $\overline{\overline{\chi}}_{\text{ee}}$, $\overline{\overline{\chi}}_{\text{em}}$, $\overline{\overline{\chi}}_{\text{me}}$, and $\overline{\overline{\chi}}_{\text{mm}}$ are the bianisotropic surface susceptibility tensors describing the metasurface.

In this work, we consider the particular case of *crossanisotropic* metasurfaces, for which $\overline{\overline{\chi}}_{\text{ee}} = \overline{\overline{\chi}}_{\text{mm}} = 0$. Under this double condition, Eqs. (1) reduce to

$$\hat{z}\times\Delta\mathbf{H} = jk\overline{\overline{\chi}}_{\text{em}}\mathbf{H}_{\text{av}}. \tag{2a}$$

$$\Delta\mathbf{E}\times\hat{z} = jk\overline{\overline{\chi}}_{\text{me}}\mathbf{E}_{\text{av}}. \tag{2b}$$

Furthermore, we introduce the following restrictions: 1) *reciprocity*, which implies $\overline{\overline{\chi}}_{\text{me}} = -\overline{\overline{\chi}}_{\text{em}}^{\text{T}}$, and hence reduces the independent susceptibility terms to the components of a single susceptibility tensor, $\overline{\overline{\chi}}_{\text{em}}$ (or $\overline{\overline{\chi}}_{\text{me}}$); 2) *transversality*, which implies that $\overline{\overline{\chi}}_{\text{em}}$ is a 2×2 (xy-plane) tensor, further restricting the number of independent tensor terms to 4; 3) *non-gyrotropy*, which implies that $\chi_{\text{em}}^{xx} = \chi_{\text{em}}^{yy} = 0$, even further restricting the independent susceptibilities to the 2 components χ_{em}^{xy} and χ_{em}^{yx}.

Finally, we shall consider a *uniform* metasurface, i.e. a metasurface whose susceptibility tensors do not depend on the position in the plane of the metasurface (χ_{em}^{xy} and χ_{em}^{yx} constant), as we wish to manipulate the magnitude and phase of the Fresnel (reflection and transmission) coefficients without altering the reflection and refraction directions dictated by conventional Snell-Descartes law.

III. Modified Fresnel Coefficients

Consider two semi-infinite media interfacing at $z = 0$, as shown in Fig. 1(a), where the medium on the left (medium 1) is air and the medium on the right (medium 2) is a dielectric with $\epsilon_{\text{r}} = 2.25$ (arbitrary numerical values chosen as an example). In such a case, the Fresnel coefficients are well-known textbook formulas [9]. The corresponding reflectance is plotted versus the incidence angle, θ_{i}, in the same figure for the case of p polarization, were the Brewster angles are referred to as θ_{B}.

We now insert our (reciprocal nongyrotropic and uniform metasurface) crossanisotropic at the interface between the two media, as shown in Fig. 1 (b). Assuming xz-plane propagation, χ_{me}^{xy} will control the response of the s, or TE, polarization and χ_{em}^{xy} that of the p, or TM, polarization.

The Fresnel coefficients for the metasurface-coated problem can be derived using Eqs. (2). This yields, under the restrictions outlined in Sec. II, the reflection (r) and transmission (t) coefficients

$$r_{\text{TM}} = \frac{\eta_1\cos\theta_1 - \eta_2\cos\theta_2\frac{(2i-k\chi_{\text{em}}^{xy})^2}{(k\chi_{\text{em}}^{xy}+2i)^2}}{\eta_1\cos\theta_1 + \eta_2\cos\theta_2\frac{(2i-k\chi_{\text{em}}^{xy})^2}{(k\chi_{\text{em}}^{xy}+2i)^2}}, \tag{3a}$$

$$t_{\text{TM}} = \frac{2\eta_2\cos\theta_1\frac{(2i-k\chi_{\text{em}}^{xy})}{(k\chi_{\text{em}}^{xy}+2i)}}{\eta_1\cos\theta_1 + \eta_2\cos\theta_2\frac{(2i-k\chi_{\text{em}}^{xy})^2}{(k\chi_{\text{em}}^{xy}+2i)^2}}, \tag{3b}$$

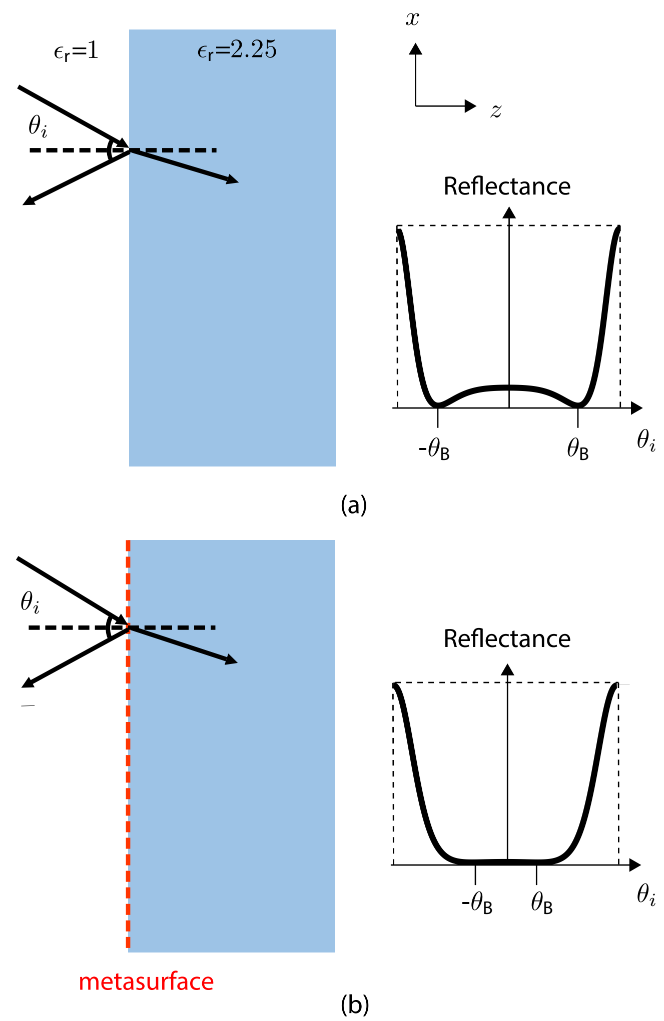

Fig. 1. Manipulation of the reflectance versus angle response of semi-infinite media boundary using metasurface coating (a) without metasurface (b) with a crossanisotropic metasurface.

$$r_{\mathrm{TE}} = \frac{\eta_2 \cos\theta_1 \frac{(2i - k\chi_{\mathrm{em}}^{yx})^2}{(k\chi_{\mathrm{em}}^{yx} + 2i)^2} - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 \frac{(2i - k\chi_{\mathrm{em}}^{yx})^2}{(k\chi_{\mathrm{em}}^{yx} + 2i)^2} + \eta_1 \cos\theta_2}, \tag{3c}$$

$$t_{\mathrm{TE}} = \frac{2\eta_1 \cos\theta_1 \frac{(2i - k\chi_{\mathrm{em}}^{yx})}{(k\chi_{\mathrm{em}}^{yx} + 2i)}}{\eta_2 \cos\theta_1 \frac{(2i - k\chi_{\mathrm{em}}^{yx})^2}{(k\chi_{\mathrm{em}}^{yx} + 2i)^2} + \eta_1 \cos\theta_2}, \tag{3d}$$

where $\theta_{(1,2)}$ and $\eta_{(1,2)}$ are the angles and the wave impedance in medium 1 and medium 2, respectively.

Comparing these relations with the the conventional Fresnel formulas suggests defining the *effective wave impedances*

$$\eta_{\mathrm{TM,eff}} = \eta_2 \frac{(2i - k\chi_{\mathrm{em}}^{xy})^2}{(2i + k\chi_{\mathrm{em}}^{xy})^2}, \tag{4a}$$

$$\eta_{\mathrm{TE,eff}} = \eta_2 \frac{(2i - k\chi_{\mathrm{em}}^{yx})^2}{(2i + k\chi_{\mathrm{em}}^{yx})^2}, \tag{4b}$$

which recast the metasurface-coated expressions to the conventional form. These effective wave impedances, depending on the susceptibilities, can then be readily exploited to set the reflection and transmission coefficients at desired values for any angle of incidence.

A particularly useful possibility is to modify the Brewster angle to any desired angle and for both polarizations [10]. The modified Brewster angles, for TM and TE polarizations, are easily found as

$$\theta_{\mathrm{B,TM}} = \arcsin\left(\sqrt{\frac{1 - (\frac{\eta_1}{\eta_{\mathrm{TM,eff}}})^2}{(\frac{n_1}{n_2})^2 - (\frac{\eta_1}{\eta_{\mathrm{TM,eff}}})^2}}\right) \tag{5a}$$

$$\theta_{\mathrm{B,TE}} = \arcsin\left(\sqrt{\frac{1 - (\frac{\eta_{\mathrm{TE,eff}}}{\eta_1})^2}{(\frac{n_1}{n_2})^2 - (\frac{\eta_{\mathrm{TE,eff}}}{\eta_1})^2}}\right). \tag{5b}$$

The reflectance in Fig. 1(a) shows a shift of the Brewster angle towards the normal, and the subsequent possibility to achieve quasi-zero reflection in a fairly large range of angles in the paraxial regime.

IV. Conclusion

The reflection and transmission properties of a *crossanisotropic* metasurface are investigated when placed at the interface between two media. It is shown that such a crossanisotropic metasurface leads to unprecedented control of the reflection and transmission coefficients. The next step of this work will be the experimental demonstration of the concept after the design of suitable scattering particles.

References

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