Rectangular Waveguide Loaded with a Dielectric Slot in a Thick Metallic Shield

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***Abstract*—Resonant transmission through a dielectric-loaded slot in a thick conducting diaphragm embedded in a rectangular waveguide is modeled using an analytical approximation based on the slot being small compared to a free space wavelength. Image theory relates this problem to a metasurface consisting of an array of such slots. We demonstrate the existence of ordinary (Fabry-Perot) transmission resonances associated with the fundamental mode of the slot, as well as extremely narrow-band (Fano) resonances involving the higher-order modes in the slot. The theoretical predictions are confirmed through full-wave numerical simulation and by experimental measurement.**

I. Introduction

Dielectric-loaded waveguides have been intensively investigated and used in many applications, for example in microwave microscopy [1], detection of pits on a metal surface [2], to name but a few. In the present study, we will investigate a structure closely related to the infinite array of slots: that of a rectangular waveguide loaded by a dielectric slot in a thick metallic screen as shown in Figure 1. By image theory, this problem can be related to that of a metasurface consisting of an array of dielectric-loaded slots in a thick conducting screen. If the incident wave in the waveguide is the fundamental TE_{10} mode, we will obtain a closed-form analytical formula to determine the transmission through this slot. Our results are validated by comparison with a full-wave numerical finite-element simulation as well as by experimental measurements.

II. Derivation

Let the width and height of the rectangular waveguide be d and L respectively. Let the thickness of the conducting shield be a, the width of the slot be $2b$, and the relative permittivity of the dielectric loading be ε_r, as shown in Figure 1.

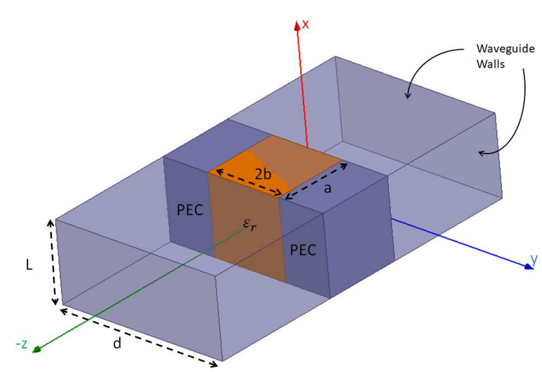

Fig. 1. Waveguide Loaded with a dielectric slot in thick metallic shield.

We formulate the problem by using a mode-matching technique [3]-[5], expanding the field E_x either as a sum of TE_{n0} waveguide modes in the large waveguide, or a sum of TE_{m0} waveguide modes in the slot:

$$E_x(y,z) = \begin{cases} A_i \cos\left(\frac{\pi y}{d}\right) e^{ik\gamma_1 z} + \sum\limits_{\substack{n=-\infty \\ n=1,3,5\ldots}}^{\infty} A_n \cos\left(\frac{n\pi y}{d}\right) e^{-ik\gamma_n z}, & z > 0 \\ \sum\limits_{m=1}^{\infty} \left[b_m e^{-ih_m(z+a)} + c_m e^{ih_m z}\right] \sin\left(\frac{\pi m}{2b}(y+b)\right), & -a < z < 0 \\ \sum\limits_{\substack{n=-\infty \\ n=1,3,5\ldots}}^{\infty} D_n \cos\left(\frac{n\pi y}{d}\right) e^{ik\gamma_n (z+a)}, & z < -a \end{cases} \tag{1}$$

We have divided our geometry to three regions—in region (I) (air, $z > 0$), the first term is the incident field, where A_i is amplitude of fundamental TE_{10} mode of the rectangular waveguide, while the summation in the second term contains the reflected TE_{n0} waveguide modes, whose amplitudes are A_n and propagation constants are $\gamma_n = \sqrt{1-(\frac{n\pi}{kd})^2}$. The even-order waveguide modes are omitted because of the symmetry of the geometry and of the incident wave. The waveguide mode expansion is the analog of the Floquet-Bloch mode expansion used in formulating the array problem, and is necessary to satisfy the boundary condition on the rectangular waveguide walls (those parallel to the $x-z$ plane in Figure 1). In region (II), $h_m = \sqrt{k^2\varepsilon_r - (\frac{\pi m}{2b})^2}$ is the propagation constant and b_m and c_m are the amplitudes of parallel-plate waveguide mode m in the slot, related to reflection and transmission coefficients at the interfaces. In region (III), we again use an expansion in modes of the large waveguide that represent the transmitted field, with mode amplitudes D_n. Next, as in [3]-[5], we apply the boundary condition $E_x = 0$ at the surface of the metallic shield and continuity of E_x and $\frac{\partial E_x}{\partial z}$ at the slot surfaces ($z = 0$ and $z = -a$). Defining $\tilde{x}_q^{\pm} = x_q^{\pm}\gamma_q$, $x_n^{\pm} = \tilde{A}_n \pm D_n$ and $\tilde{A}_n = A_n$ if $n \neq 1$ and $\tilde{A}_1 = A_i + A_1$ if $n = 1$, we arrive at the equations:

$$\tilde{x}_q^{\pm} = \frac{-2k\gamma_q}{\pi^2 b^3 d} \cos\left(\frac{q\pi b}{d}\right) \times$$

$$\times \left[2\cos\left(\frac{\pi b}{d}\right) \sum_{m=1,3,5\ldots.}^{\infty} \frac{m^2 \frac{\Gamma_m^{\pm}}{\Gamma_m^{\mp}}}{h_m \left[\left(\frac{q}{d}\right)^2 - \left(\frac{m}{2b}\right)^2\right] \left[\left(\frac{1}{d}\right)^2 - \left(\frac{m}{2b}\right)^2\right]} \right.$$

$$-\sum_{m=1,3,5....}^{\infty}\frac{m^2\frac{\Gamma_m^{\pm}}{\Gamma_m^{\mp}}}{h_m\left[\left(\frac{q}{d}\right)^2-\left(\frac{m}{2b}\right)^2\right]}\sum_{\substack{n=-\infty\\n=1,3,5...}}^{\infty}\frac{\tilde{x}_q^{\pm}\cos\left(\frac{n\pi b}{d}\right)}{\left[\left(\frac{n}{d}\right)^2-\left(\frac{m}{2b}\right)^2\right]}\Bigg] \tag{2}$$

where $\Gamma_m^{\pm}=e^{-ih_m a}\pm 1$. If we assume that $k\sqrt{\varepsilon_r}<\frac{5\pi}{2b}$, then only the first two modes will propagated ($m=1$ and 3) and other higher-order parallel plate waveguide modes ($m>3$) will be evanescent. As a first approximation, we will eliminate all evanescent modes in the slot, obtaining from equation 2 a finite system whose solution is:

$$\tilde{x}_1^{\pm}=\frac{-4kb\gamma_1\pi^2}{d}\left[\frac{1}{h_1}\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}G_{11}^2\left(\frac{\pi b}{d}\right)+\frac{9}{h_3}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}G_{33}^2\left(\frac{\pi b}{d}\right)\right]$$

$$+C_1^{\pm}\gamma_1 G_{11}\left(\frac{\pi b}{d}\right)+C_3^{\pm}\gamma_1 G_{33}\left(\frac{\pi b}{d}\right) \tag{3}$$

where $G_{11}(x)=\frac{\cos(x)}{\left[(x)^2-\left(\frac{\pi}{2}\right)^2\right]}$, $G_{33}(x)=\frac{\cos(x)}{\left[(x)^2-\left(\frac{3\pi}{2}\right)^2\right]}$, $C_1^{\pm}$ and $C_3^{\pm}$ are defined in (4) and (5) respectively, and,

$$I_{11}=\sum_{\substack{n=-\infty\\n=1,3,5...}}^{\infty}\gamma_n G_{11}^2\left(\frac{n\pi b}{d}\right), I_{33}=\sum_{\substack{n=-\infty\\n=1,3,5...}}^{\infty}\gamma_n G_{33}^2\left(\frac{n\pi b}{d}\right)$$

$$I_{13}=\sum_{\substack{n=-\infty\\n=1,3,5...}}^{\infty}\gamma_n G_{11}\left(\frac{n\pi b}{d}\right)G_{33}\left(\frac{n\pi b}{d}\right)$$

If only the fundamental mode ($n=1$) of the rectangular waveguide is assumed to propagate, from (3) we can determine the magnitude of the plane-wave transmission coefficient:

$$|S_{12}|=\left|\frac{E_{xt}(y,z)}{E_{xi}(y,z)}\right|=\left|\frac{D_1\cos\left(\frac{\pi y}{d}\right)e^{ik\gamma_1(z+a)}}{\cos\left(\frac{\pi y}{d}\right)e^{ik\gamma_1 z}}\right|=|D_1| \tag{6}$$

III. Results

We consider WR-112 waveguide ($d=28.4988$ mm and $L=12.6238$ mm, frequency range 7-10 GHz) and slot of following dimensions: thickness $a=5.6$ mm and width $2b=10.5$ mm, loaded with a material for which $\varepsilon_r=73.5$ and loss tangent 1×10^{-3} and 8331-silver conductive epoxy adhesive (conductivity $\sigma\simeq 14.3kS/m$) used to glue the dielectric to waveguide section as shown in Figure 2.

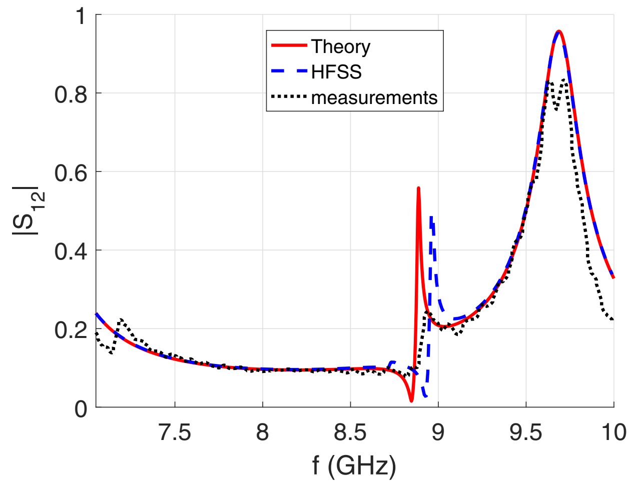

Fig. 2. $|S_{12}|$ calculated from (6) for WR-112 rectangular waveguide compared with HFSS and measurement results: $a=5.6$ mm, $2b=10.5$ mm and $\varepsilon_r=73.5(1-i10^{-3})$

The results show very good agreement between analytical and numerical results, while the disagreement in the measurement result is due to the losses in the 8331-silver conductive epoxy adhesive that fill the gap between the dielectric and the waveguide section.

References

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$$C_1^{\pm}=\frac{-8\left(\frac{\pi^2kb}{dh_1}\right)^2\left(\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}\right)^2G_{11}\left(\frac{\pi b}{d}\right)I_{11}-72\frac{\pi^4k^2b^2}{d^2h_1h_3}\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}G_{33}\left(\frac{\pi b}{d}\right)I_{13}-144\frac{k^3b^3\pi^6}{d^3h_1^2h_3}\left(\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}\right)^2\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}G_{11}\left(\frac{\pi b}{d}\right)\left[I_{11}I_{33}-I_{13}^2\right]}{\left[1-\frac{2\pi^2kb}{dh_1}\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}I_{11}-\frac{18kb\pi^2}{dh_3}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}I_{33}+36\frac{\pi^4k^2b^2}{d^2h_1h_3}\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}\left[I_{11}I_{33}-I_{13}^2\right]\right]} \tag{4}$$

$$C_3^{\pm}=\frac{-72\frac{k^2b^2\pi^4}{d^2h_1h_3}\frac{\Gamma_1^{\pm}}{\Gamma_1^{\mp}}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}G_{11}\left(\frac{\pi b}{d}\right)I_{13}-648\left(\frac{kb\pi^2}{dh_3}\right)^2\left(\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}\right)^2G_{33}\left(\frac{\pi b}{d}\right)I_{33}+C_1^{\pm}\frac{18kb\pi^2}{dh_3}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}I_{13}}{\left[1-\frac{18kb\pi^2}{dh_3}\frac{\Gamma_3^{\pm}}{\Gamma_3^{\mp}}I_{33}\right]} \tag{5}$$