Simplifying and Generalizing Antenna Array Expressions with the Antenna Equation

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When describing antenna performance in a phased array, one needs to keep track of both magnitude and phase. Antenna gain and realized gain are insufficient, since they lack phase information. To get around this, one typically uses effective length, which is the ratio of the received open circuit voltage to the incident electric field (IEEE, *IEEE Standard Definitions of Terms for Antennas*, IEEE Std 145™–2013, Institute for Electrical and Electronics Engineering, New York, December 2013). However, effective length has some problems and limitations.

The first problem with effective length is that equations based on it are unnecessarily complicated. In the frequency domain, this is merely inconvenient, but in the time domain this conceals essential features of antenna performance.

A second problem with effective length is that it is difficult to use with waveguide feeds, because one cannot easily measure an open circuit voltage across a waveguide. One can avoid the problem by adding waveguide-to-coax adapters to the antenna ports, but the adapter response is then mixed in with the antenna response. And if waveguide circuitry is added to the antenna port, it is impossible to separate the antenna response from that of the circuit. It would be preferable to have a set of equations that handle waveguide feeds more naturally.

We address these issues with the newly developed antenna equation (E.G. Farr, "Characterizing Antennas in the Time and Frequency Domains," *IEEE Antennas and Propagation Magazine*, February 2018, pp 106-110.) and (E. G. Farr, "A Power Wave Theory of Antennas," *Forum for Electromagnetic Methods and Application Technologies* (online), Vol. 7, 2015, www.e-fermat.org). The antenna equation describes antenna performance in a manner that is both compact and elegant. It works in both the time and frequency domains, and in both transmission and reception. It also adds a meaningful phase to antenna gain and radar cross section. By adapting this to antenna arrays we achieve a significant simplification of the expressions used to describe antenna arrays, and we generalize the results to waveguide feeds.

An added benefit of using the antenna equation is that it shows how to generalize mutual impedance in an antenna array. Mutual impedance in an antenna array is the ratio of the open-circuit voltage produced at one terminal to the current supplied to a second terminal, when all other terminal pairs are open-circuited. This is undefined for waveguide feeds, because one cannot easily measure an open-circuit voltage across a waveguide. As before, one can add waveguide-to-coax adapters to all the ports, but the adapter response is then included with the antenna response. The antenna equation allows one to isolate the mutual coupling response from the adapter, using simpler equations than those used previously. This results in a new concept, a unique and well-defined "mutual coupling coefficient" in antenna arrays. In the time domain, this becomes the "mutual coupling impulse response."