Dressed state approach to quantum electromagnetics

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***Abstract*—We briefly define the scope of the field of quantum electromagnetics and introduce the dressed state approach to tackle problems in this field, such as atom-field interaction, multi-atom coherence as well as the Casimir-Polder interaction amongst atoms and their electromagnetic environments. In particular we emphasize how classical computational electromagnetic knowledge is injected into these quantum mechanical calculations. The principle results of our efforts are summarized and a lookout for future studies are presented.**

I. Introduction

Recent advances in circuit quantum-electrodynamics (C-QED) have reinvigorated interests in the theoretical models of QED [1]. Since relativistic effects and the internal structures of the atoms are both unimportant for the artificial atoms used in C-QED, one is prompted to shift away from conventional QED in modeling these systems. Instead, the complicated microwave circuits in which these artificial atoms operate invites the computational electromagnetics (CEM) community onto the scene. This special interplay between simplified atomic models and complex EM environments have been termed quantum electromagnetics (QEM) [2], [3].

In this summary paper we present our group's recent progress in the field of QEM, especially in connection with the dressed state approach [4]–[8]. In Section II the pertinent equations of QEM are given. In Section III the dressed state approach is reviewed and our main results stated. In Section IV the dressed states solutions are connected with the Casimir Polder interaction. Finally, Section V presents our goal of using the dressed state approach to extend QEM to treat dissipative quantum systems [9].

II. Equations of quantum electromagnetics

Quantization of EM fields have long been studied [1], [6]. However, the conventional approach relies on a mode decomposition method that may prove costly for complex EM environments and altogether impossible for dissipative ones. In [2], [3], an alternative field quantization approach in arbitrary lossless media is presented without using mode decomposition. This approach is akin to the correspondence principle between classical and quantum mechanical systems.

For *external* sources the equations of QEM are [3]:

$$\nabla\times\hat{\mathbf{H}}(\mathbf{r},t)-\partial_t\hat{\mathbf{D}}(\boldsymbol{r},t)=\hat{\mathbf{J}}_{\rm ext}(\boldsymbol{r},t),\qquad \nabla\cdot\hat{\mathbf{D}}(\boldsymbol{r},t)=\hat{\rho}_{\rm ext}(\mathbf{r},t)$$
$$\nabla\times\hat{\mathbf{E}}(\mathbf{r},t)+\partial_t\hat{\mathbf{B}}(\mathbf{r},t)=0,\qquad \nabla\cdot\hat{\mathbf{B}}=0 \tag{1}$$

In this external source regime it was shown that the classical dyadic Green's function completely captures the radiation of the sources into quantized fields [3]. The full dynamical problem of QEM, i.e. the one with *internal* sources, is attained if in addition to (1) the equations of motion of the source driven by the quantized fields are given. In the next section we discuss the solution of an important class of this system using the dressed state approach, and focus on how the classical dyadic Green's function can be used in the solution.

III. Dressed state approach

The dressed state approach is similar in spirit to diagonalizing a matrix system [4]. It is by no means a new method in treating QED problems [6]–[8]. The model Hamiltonian is:

$$\hat{H}=\sum_n\omega_n\,\hat{b}_n^\dagger\hat{b}_n+\sum_{\mathbf{k}}\omega_{\mathbf{k}}\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}}$$
$$+\sum_{n,\mathbf{k}}ig_{n,\mathbf{k}}(\hat{b}_n^\dagger+\hat{b}_n)(\hat{a}_{\mathbf{k}}-\hat{a}_{\mathbf{k}}^\dagger) \tag{2}$$

Here the $\hat{b}_n,\hat{b}_n^\dagger$ represent atomic excitations, which may obey fermion or boson commutation relations, corresponding to few-level or harmonic oscillator atoms, respectively [4]. The operators $\hat{a}_{\mathbf{k}},\hat{a}_{\mathbf{k}}^\dagger$ represent field excitations, or photons, which are addressed by their wavenumber number $\mathbf{k}$. The term in the second line of (2) represents the full quantized dipole interaction[1] $\hat{\mathbf{E}}(\mathbf{r}_n)\cdot\hat{\mathbf{d}}_n$. The Hamiltonian operator in (2) is a mode decomposed representation of the system in (1), albeit with internal sources that are highly simplified.

The goal of the dressed state approach is to find *linear* combinations of the original atomic and field excitations (or de-excitations) that are non-interacting, and hence evolve in time as time harmonics. For harmonic oscillator atoms, the dressed operators have the following expansion:

$$\hat{d}_i=\sum_n A_{i,n}\hat{b}_n+\sum_n B_{i,n}\hat{b}_n^\dagger+\sum_{\mathbf{k}}F_{i,\mathbf{k}}\hat{a}_{\mathbf{k}}+\sum_{\mathbf{k}}K_{i,\mathbf{k}}\hat{a}_{\mathbf{k}}^\dagger \tag{3}$$

The dynamics of the system are then posed in terms of the transformation between observables and these dressed state operators. Such a solution of the dynamics is non-perturbative. The problem is in fact closely related to classical coupled

[1]When the product of operators is expanded in (2), two of the resulting terms involve one creation and one annihilation operator, these are termed *rotating wave* terms, while the other two are the *counter rotating wave* terms.

mode theory. However, the counter rotating wave terms in the dipole interaction produces some rather non-classical effects.

For few-level atoms, the dressed state solution is not as straightforward. One must separate the dynamics into different excitation subspaces [4]. Exact solutions are only possible under the rotating wave approximation, counter-rotating wave terms must be treated using perturbation theory [11].

The use of the mode decomposition picture is problematic in light of our previous discussion. However, it is only a temporary measure as all quantities of interest can be related to the dyadic Green's function through the relation [4], [5]:

$$\Gamma_{nm}(\omega) = \frac{k^2}{\varepsilon\,\pi}\,\mathbf{d}_n \cdot \mathrm{Im}\{\bar{\mathbf{G}}_E(\mathbf{r}_n, \mathbf{r}_m; \omega)\} \cdot \mathbf{d}_m \qquad (4)$$

Here, Γ_{nm} is the cooperative decay rate between the n-th and m-th atomic excitation and $\bar{\mathbf{G}}_E$ is the electric dyadic Green's function. The quantities $\mathbf{d}_n, \mathbf{d}_m$ are the dipole moments of the atoms. The dressed state approach allows one to write down the solution to the dynamics in terms of Γ_{nm}. Relation (4), in turn, means the solution is fully dependent on the numerical evaluation of the classical dyadic Green's function [4], [5]. Therefore, a full CEM based solution procedure is established for an important class of QEM problems.

Using this solution procedure we have been able to calculate the following in arbitrary *lossless* EM environments.

1) Spontaneous emission and single photon scattering from a single atom [4].
2) Super- and sub-radiance of multiple atoms [5].
3) Atom-photon bound states analogous to Anderson localization in solid state systems [4].
4) Tight-binding behavior of multiple atom-photon bound states [5].

IV. Casimir-Polder interaction

The Casimir-Polder interaction between atoms and conducting surfaces is one of the most famous predictions of QED. The experimental observation of this interaction proves the existence of the quantum vacuum. Previously our group had performed calculations of the Casimir force between conducting objects using CEM techniques [12], [13]. The dressed state approach allows us also to calculate the Casimir-Polder interaction between atoms and surfaces, and among atoms.

A dressed state calculation of the Casimir-Polder interaction for harmonic oscillator atoms and few-level atoms have been done in [10] and [11], respectively. We will present the connection of the atom-photon bound state energy reported in [4], [5] with the Casimir-Polder interaction between atoms and surfaces, as well as among atoms. In particular, the distinction of the Casimir-Polder energy with and without the rotating wave approximation will be made.

V. Dissipative quantum electromagnetics

Quantum dissipation is a difficult problem since loss cannot be easily incorporated into a quantum system, as opposed to its classical counterpart [14]. However, quantum dissipation is becoming more important with the advances in quantum computing and quantum information processing hardware. A correct treatment of quantum dissipation is vital to the development of next generation quantum technologies.

The main challenge in dissipative quantum systems is the preservation of equal time commutator for the operators of interest. This is equivalent to ensuring that the total quantum system is Hermitian and hence energy conserving [9]. In the regime of QED and QEM, dissipative quantum systems were first studied in the quantization of electromagnetic fields in dielectrics [6], the phonons in the dielectric being the source of dissipation. A dressed state approach was employed, in which the coupled photon and phonon excitations, termed polaritons, are found to diagonalize the total Hamiltonian. Though containing good physical insight, the polariton operators are rather unwieldy for actual calculations. The alternative approach is to introduce Langevin sources for the field operators [9], [14].

We propose to use the dressed state approach to connect the quantum dissipation with the classical, lossy dyadic Green's function. Early attempts at this connection will be presented in the context of a one dimensional system of atoms, photons and phonons. Such a system, though simple, is relevant to experiments in circuit QED where a microwave transmission line connects several superconducting qubits.

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