Accuracy Study of Singularity Extraction Method for Near-Singular and Near-Hypersingular Surface Integrals in Higher Order Method of Moments

Sanja B. Manić*[1], Ana B. Manić[1], and Branislav M. Notaroš[1]
(1) Colorado State University, Department of Electrical and Computer Engineering, Fort Collins, CO, USA, smanic@engr.colostate.edu, manic.ana@gmail.com, notaros@colostate.edu

This paper presents our ongoing study related to convergence behavior of near-singular (potential) and near-hypersingual (field) integrals for double higher order large-domain method of moments in the surface integral equation formulation (MoM-SIE). The importance of fast and accurate integral evaluation in computing the MoM matrix entries is essential in MoM-SIE modeling and applications, when small source-to-field distances occur, i.e., when testing and basis element are close to each other. A typical application when this comes out to be especially critical and can enhance the accuracy and speed up the analysis is, for example, microstrip and printed circuit design.

In this study, the integrals are defined on the Lagrange-type generalized curved parametric quadrilateral MoM-SIE surface elements of first and second geometrical orders with the unknown surface current being approximated by polynomial basis functions of higher orders (up to the third order). The technique for integral evaluation presented here uses the singularity extraction method, which evaluates analytically and numerically an integral of a function with similar singularity as the original function and adds the difference to the original integral that is evaluated numerically.

The analytically computed integral of the principal singular part is computed over the parallelogram whose surface is defined to be similar to the surface of the generalized quadrilateral near the singular point. Projection of the field point to the quadrilateral element is found in order to compute parameters defining the parallelogram. Near-singular and near-hypersingular integrands over the parallelogram are represented through Taylor or shifted Taylor series of the original functions. Numerical integrals over the parallelogram and over the generalized quadrilateral are computed using Gauss-Legendre quadrature formulas.

In our approach, analytical integrals over a triangle are computed using recursive formulas in the similar manner to the procedure described by S. Jarvenpaa, M. Taskinen, and P. Yla-Oijala in 2006 ("Singularity subtraction technique for high-order polynomial vector basis functions on planar triangles," IEEE Trans. Antennas Propag., vol. 54, pp. 42-49). Four triangles are obtained, each by connecting the projection point, which is in the same plane as a parallelogram, to one of four parallelogram sides. The referenced method was expanded to analyse integrals with higher order polynomial basis functions defined over a parallelogram.

We investigate convergence of the novel integration method with increasing the orders of Gauss-Legendre integration formulas, i.e., numbers of integration points, over quadrilateral patches, in a variety of numerical examples. We compare the new method with a previously implemented extraction technique in the double higher order MoM-SIE framework and discuss the accuracy of the computation of individual near-singular and near-hypersingual integrals, as well as the overall MoM-SIE solution accuracy in scattering examples.