Phase Response at Resonance Frequency for Metamaterial-Insert Mediums

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***Abstract*— The phase propagation across the metamaterial - insert is essential to characterize the medium as epsilon-negative (ENG), mu-negative (MNG) or double negative (DNG). In this paper, we study the phase behavior on propagating through metamaterial-insert. For ENG or MNG media, we find that the phase propagation through the metamaterial insert has zero slope at the resonant frequency and positive slope at other frequencies. In other words, the medium has a band-gap at resonance, which results in constant phase across the medium. For the DNG media, the phase propagation through metamaterial-insert has negative slope at the resonance frequency and positive slope at other frequencies. In other words, the propagation of the wave is backward at the resonance which results in negative phase slope across the medium.**

***Index Terms*— Double negative (DNG), epsilon-negative (ENG), metamaterial-insert, mu-negative (MNG), phase propagation.**

I. INTRODUCTION

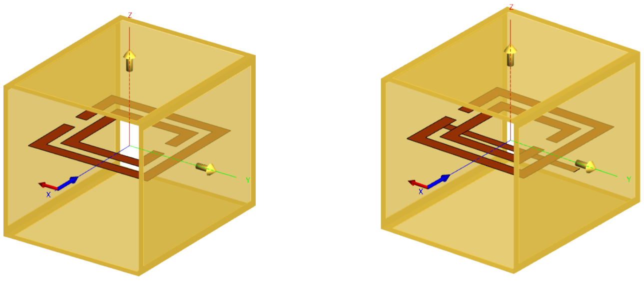
CHARACTERIZATION of materials for permittivity, ε, and permeability, μ, can be performed using scattering matrix parameters as the input in the retrieval process [1], [2], [4]. For the lossless conventional media (ε and μ are positive), the wave number, β, for a given electromagnetics field is also positive. For the single negative (SNG) media (either epsilon negative (ENG) or mu-negative (MNG)), β becomes imaginary number which presents the evanescent waves. For double negative (DNG) media, the real part of β is negative which presents the backward-waves.

However, the bulk effective parameters (permittivity, ε, permeability, μ, refractive index, n) may not be retrieved correctly which often lead to wrong classification of materials [3]. In this paper, we analyze the phase propagation across the metamaterial-insert medium. This can be used to classify the medium as single negative (SNG) (epsilon-negative (ENG) or mu-negative (MNG)) or double negative (DNG).

The metamaterial inserts selected in this study are the split-ring resonator (SRR) and the continuous wire (CW). The SRR arranged in the periodic lattice presents SNG media and the combination of SRR and CW presents DNG media. To simplify the problem, the relative permittivity ε_r of the substrate carrying the ring, loop, and wire is set to 1 for free space. The incident plane wave's electric field is polarized in the direction of the wire in the metamaterial. The phase propagation across media is simulated using the commercial electromagnetic simulator, FEKO.

II. COMPLEX WAVE NUMBER OF SRR AND SRR PLUS CW INSERT

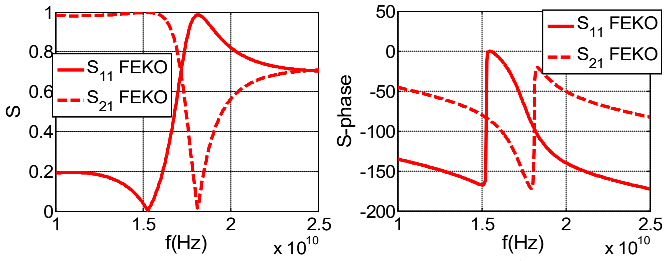
In this study, we choose SRR and SRR plus CW as the insert to simulate SNG medium and DNG medium, respectively. The geometry and dimensions of the inserts are the same as the design proposed by Smith *et al* in [4]. The unit cell is placed at the origin and excited by a y-polarized electric field in a plane wave traveling along the xz-plane, as shown in Figure 1. This is referred to as parallel incidence, where the material extends to infinite in the *y* and *z* directions, with thickness of one cells in the *x*-direction.



(a) (b)

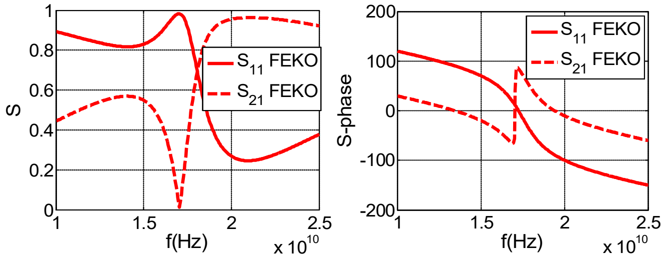
Figure 1. Unit cell of a metamaterial-insert consisting of (a) SRR and (b) SRR plus continuous wire

The S parameters for the unit cell of Figure 1a and Figure 1b are computed using FEKO, as shown in Figure 2 and 3 for both amplitude and phase respectively. The flip in the phase of S_{21} indicates the presence of a the resonant region (around 17GHz). Using S parameters as the input, the complex wave number β of the metamaterial-insert in Figure 1 can be extracted based on the Nicolson-Ross-Weir concept [1]-[2], as shown in Figure 4.



(a) (b)

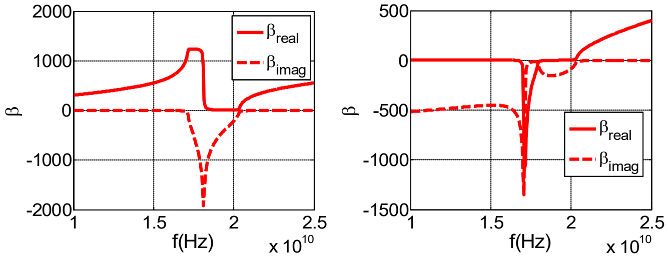
Figure 2. (a) Magnitude and (b) phase of simulated S-parameters for the unit cell in Figure 1a.



(a) (b)

Figure 3. (a) Magnitude and (b) phase of simulated S-parameters for the unit cell in Figure 1b.

As we can see, Figure 4a shows a bandgap region of zero wave propagation (β becomes imaginary number) described as a SNG behavior which presents the evanescent waves. Figure 4b shows a region of negative wave propagation descried as a DNG behavior which presents the backward-waves.



(a) (b)

Figure 4. Extracted real and imaginary of β for (a) the unit cell in Figure 1a, (b) the unit cell in Figure 1b.

III. PROPAGATION PHASE STUDY OF SRR AND SRR PLUS CW INSERT

To verify SNG and DNG behavior as suggested from the complex wavenumber β as shown in Figure 4, we analyze the phase profile in the direction of propagation (x-axis). The observation line is constructed so that its length in x-axis is three unit cells and metamaterial-insert is placed in the middle. A y-polarized E-field plane wave traveling along x-axis is used as the excitation to the unit cell.

Figure 5 shows the phase of the electric field (y component) along x-axis at 10 GHz and 17.9 GHz for the metamaterial-insert described in Figure 1a. At 10 GHz (non-resonant frequency), the phase front inside the metamaterial-insert as well as the region before and after waves enter and exit the insert have the same positive slope, as depicted in the blue curve. This is indicating that the metamaterial-insert acts as conventional positive dielectric material, or free space in this example. This is also corresponding to β positive. At 17.9GHz (the resonant frequency), as seen in the green curve , the phase front inside the metamaterial-insert has a zero slope, which can be interpreted as the band-gap in that region. This is also matching to imaginary number, β.

Similarly, we simulate the phase of the electric field (y component) along x-axis at 10 GHz and 17.4 GHz for the insert described in Figure 1b, as depicted in Figure 6.

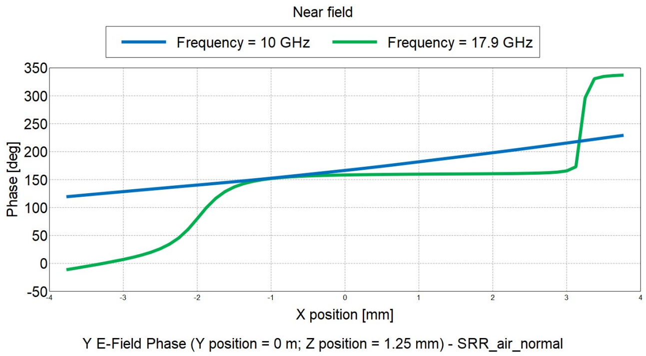


Figure 5. The phase profile of the electric field (y component) for the metamaterial-insert described in Figure 1a at 10 GHz and 17.9GHz.

As we observe, the slope of phase inside metamaterial is negative at the resonance frequency. The negative slope also verifies that the real part of β is negative which presents the backward-waves in this region.

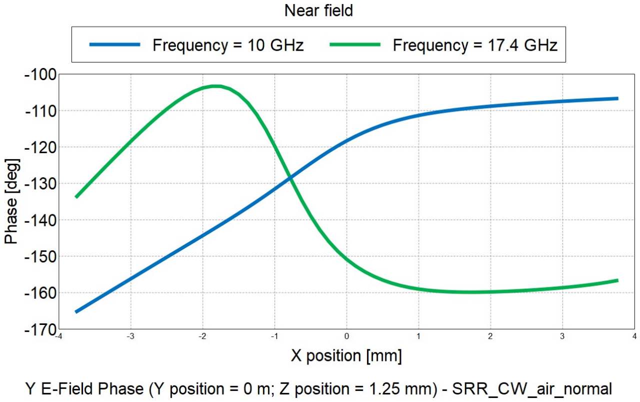


Figure 6. The phase profile of the electric field (y component) for the metamaterial-insert described in Figure 1b at 10 GHz and 17.4GHz.

IV. CONCULSION

In this work, we studied the phase front when the waves propagate through a metamaterial-insert. For SNG medium, we find that the phase front has a zero slope in the region inside the metamaterial-insert at the resonance frequency, which can be interpreted as band-gap region in the frequency band. For DNG medium, the phase front has a negative slope inside metamaterial-insert at the resonance frequency, which can be depicted as the backward-waves in this region.

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