All-pass Characteristics of a Huygens' Unit Cell

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***Abstract*—Huygens' unit cells have demonstrated great versatility and they are used in a widespread of applications for their ability to manipulate electromagnetic wavefronts at will. Common applications include: electromagnetic-wave refraction, focusing, defocussing and beam conversion. They can achieve these multitude of applications with no or extremely low reflection losses. This paper investigates the possibility of also achieving a wide matching bandwidth from a Huygens' unit cell. This is enabled by leveraging on the equivalence between Huygens' unit cells and lattice networks which in turn can be designed for an all-pass response. Based on this equivalence, conditions on achieving a similar all-pass response from a Huygens' unit cell are derived. A case study of a wire-loop Huygens' unit cell is presented validating these conditions with full-wave analysis and providing an extremely wide matching bandwidth.**

I. Introduction

Huygens' metasurfaces can be engineered to manipulate electromagnetic wavefronts at will [1], [2]. They can be designed for many applications such as electromagnetic-wave refraction, focusing, defocussing and beam conversion among other applications [3]. They comprise electric and magnetic current sheets that are suitably weighted by the incident field depending on the application of interest. Typically, these current sheets are implemented as passive surface impedances [1], [2]. However, active implementations of these current sheets have also been proposed in the literature [2].

This paper investigates the characteristics of the matching bandwidth of Huygens' unit cells by leveraging on the equivalence between them and lattice networks [4]. It is well established that lattice networks of different orders can achieve an all-pass response. Hence, this paper poses the question whether or not an equivalent Huygens' unit cell can also provide an all-pass response. Such an all-pass type of response would achieve the best possible matching bandwidth.

II. Equivalence between a Huygens' Unit Cell and a Lattice Network

A general depiction of a Huygens' unit cell and a lattice network are shown in Fig. 1. An equivalence between them has been established in [4]. The equivalence shown is between the E/H boundary conditions across a Huygens' unit cell and the terminal V/I relations across a lattice network where the two relations have a one-to-one correspondence. It can be stated as $Z_1 = Z_m/2$ and $Z_2 = 2Z_e$ where Z_m/Z_e are the magnetic/electric surface impedances of the wires/loops and Z_1/Z_2 are the series/shunt impedances of the lattice network.

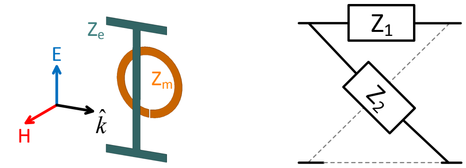


Fig. 1. A Huygens' unit cell and the equivalent lattice network.

This established equivalence raises the important question whether or not it is possible to engineer a Huygens' source that is all-pass in nature similar to an all-pass lattice network.

III. Theory for an All-pass Response from a Lattice Network

The S-matrix of the lattice network in Fig. 1, referenced to a port impedance of Z_o, is given by:

$$[\boldsymbol{S}]_{\boldsymbol{LN}} = \frac{1}{\Delta Z}\begin{bmatrix} Z_1 Z_2 - Z_o^2 & Z_o\left(Z_2 - Z_1\right) \\ Z_o\left(Z_2 - Z_1\right) & Z_1 Z_2 - Z_o^2 \end{bmatrix} \tag{1}$$

where $\Delta Z = \left(Z_1 + Z_o\right)\left(Z_2 + Z_o\right)$. An all-pass response from a lossless network requires $S_{11} = S_{22} = 0$ and $S_{12} = S_{21} = e^{j\phi}$ where ϕ is the required phase delay. Applying these two requirements on $[\boldsymbol{S}]_{\boldsymbol{LN}}$ results in the following conditions on Z_1 and Z_2:

$$Z_1 Z_2 = Z_o^2, \quad \frac{\sqrt{Z_2} - \sqrt{Z_1}}{\sqrt{Z_2} + \sqrt{Z_1}} = e^{j\phi} \tag{2}$$

The first condition guarantees the all-pass response whereas the second controls the phase delay provided that the first one is satisfied.

IV. Investigation of All-pass Response of a Wire-Loop Huygens' Unit Cell

The equivalence between the lattice network and the Huygens' unit cell can be applied to the conditions in (2). This results in the following conditions on the surface impedances of the Huygens' unit cell:

$$Z_e Z_m = Z_o^2, \quad \frac{2\sqrt{Z_e} - \sqrt{Z_m}}{2\sqrt{Z_e} + \sqrt{Z_m}} = e^{j\phi} \tag{3}$$

It is observed from the first condition that the electric and magnetic surface impedances Z_e and Z_m are required to be duals to each other at all frequencies and balanced to Z_o for a true all-pass response to be achieved.

It is proposed to employ a wire-loop pair as a Huygens' unit cell and operate them around their first resonances. The first resonance of a finite wire is a series resonance which can be modeled as $Z_e = j\omega L_e - j/(\omega C_e)$ as depicted in Fig. 2a. On the other hand, the first resonance of a slotted loop is a shunt resonance which can be modeled as $1/Z_m = j\omega C_m - j/(\omega L_m)$ as shown in Fig. 2b.

A Huygens' unit cell operated at the first resonances of the wire-loop pair can be modeled as a second order lattice network as described by Fig. 2c. It is important to highlight that the wire-loop pair can exhibit higher order resonances that will perturb the required all-pass response at higher frequencies and are expected to limit the bandwidth of operation. Nevertheless, a significant bandwidth of operation is expected, provided that the conditions of (3) are satisfied. Substituting the second order lattice network model into the conditions described in (3) results in:

$$\sqrt{\frac{L_e}{C_m}} = Z_o, \quad \omega_o = \frac{1}{\sqrt{L_e C_e}} = \frac{1}{\sqrt{L_m C_m}}$$
$$\frac{2j\omega\sqrt{L_e C_m}\left(1-\omega_o^2/\omega^2\right)-1}{2j\omega\sqrt{L_e C_m}\left(1-\omega_o^2/\omega^2\right)+1} = e^{j\phi} \tag{4}$$

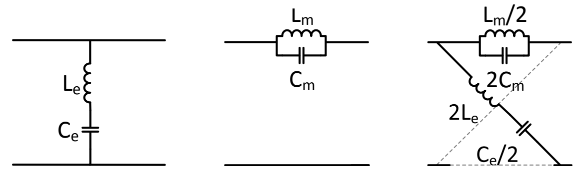
where ω_o is the resonance frequency of the superimposed wire-loop pair. Thus, if the Huygens' unit cell is designed so that the first resonances of the wire-loop pair coincide in frequency, the duality condition is directly satisfied in the vicinity of the resonance frequencies. In addition, if the two resonators are balanced to Z_o, an all-pass response is expected.

A dogbone wire was simulated around its first resonance frequency using the full-wave simulator HFSS. The S-parameters response including copper losses are depicted in Fig. 3a (referenced to 377Ω). Similarly, a loop with a slit was simulated using HFSS around its first resonance frequency. The S-parameters response including copper losses are shown in Fig. 3b (referenced to 377Ω). It is observed that there is a higher order resonance that is expected to perturb the Huygens' unit cell operation at higher frequencies.

The wire and loop were superimposed and the pair is modeled as described by Fig. 2c. There was a slight change in the extracted parameters values that can be attributed to mutual coupling between the wire and loop. The full-wave simulated S-parameters response is shown in Fig. 3 (referenced to $\sqrt{L_e'/C_m'} = 97\Omega$). Indeed, an all-pass type of response is observed for an extremely wide bandwidth of operation. This all-pass response is limited at higher frequencies by the higher order resonances of the wire-loop pair. Nevertheless, if these resonances are also engineered to be duals and coinciding in frequency, the conditions in (3) will be satisfied. This can potentially extend the matching bandwidth even further.

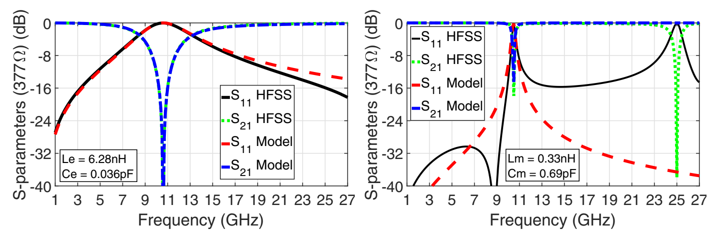
It is also observed that the all-pass type of response was achieved for a wave-port impedance of 97Ω. A higher wave-port impedance could be achieved by loading the wire-loop pair with reactances and/or optimizing the dimensions and shape of the pair.

It should be pointed out that a pair of dogbones can provide a similar frequency response. However, the electric and magnetic resonances are hard to tune independently to achieve an all-pass response [5].

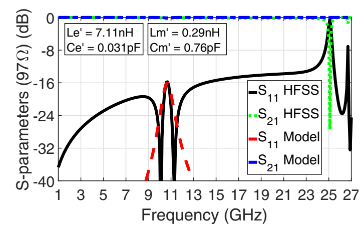


(a) Finite Wire (b) Loop (c) Wire-Loop Pair

Fig. 2. Circuit model of a finite wire, a loop and a wire-loop pair.



(a) Finite Wire (b) Loop



(c) Wire-Loop Pair

Fig. 3. S-parameters of a finite wire, a loop and a wire-loop pair generated using Floquet ports and periodic boundary conditions.

V. Conclusion

In this paper, conditions on achieving an all-pass response from Huygens' unit cells have been derived. These are simply achieving a dual response between the electric/magnetic surface impedances and balancing them for a particular wave-port impedance. This was demonstrated by employing a wire-loop Huygens' unit cell. The full-wave simulations demonstrated that an all-pass response is possible with a wide matching bandwidth.

References

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