Transmission Through an Inhomogeneous Dielectric-Loaded Slot in an Infinite Metallic Shield of Finite Thickness

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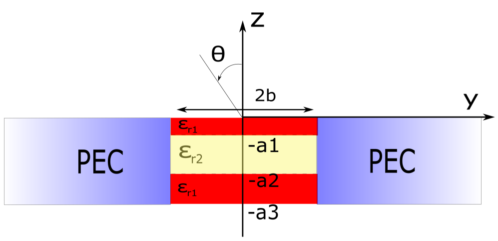

Fig. 1. A metallic shield with an inhomogeneously filled slot.

***Abstract*—Extraordinary transmission through an inhomogeneous dielectric loaded slot in a an infinite metallic shield of finite thickness is demonstrated. We show that frequency intervals between resonances can be controlled by introducing a region of different dielectric constant within the slot.**

I. Introduction

Transmission through an infinite dielectric-loaded slot in an infinite metallic shield of finite thickness has been intensively studied by many authors [1]-[8]. Understanding the single element behavior is vital to understanding the behavior of a periodic array of such slots, such as might serve as an array antenna, or (nowadays) as a metasurface [9]. Such a structure has been treated previously using an integral equation with a mode-series expansion [1], [5], [7], [8], and via an equivalent circuit admittance model [2]-[4]. Most previous works have focused their attention only on the case where the slot is homogeneously filled with dielectric. In this paper, we introduce a region of different dielectric permittivity within the slot, and obtain an analytical formula for the transmission factor of the slot. Our analysis reveals that the thickness and location of the gap within the slot, as well as the dielectric constant of the substance that fills the gap, can control the separation between resonances.

II. Derivation

Let an H-polarized electromagnetic wave be obliquely incident at an angle θ to a single inhomogeneous dielectric-loaded slot of width $2b$ in a perfectly conducting metallic shield of finite thickness a as shown in Figure 1. The slot consists of three layers of different thicknesses where 1st and 3rd layers of thickness a_1 and $a_3 - a_2$ respectively are filled by a medium with relative permittivity ε_{r1} and the layer in the middle of thickness $a_2 - a_1$ is filled by one with ε_{r2}. We begin with an integral equation formulation with a mode-series expansion as used in [1]. The magnetic field is expressed as:

$$H_x(y,z) = \begin{cases} e^{-ik(\alpha y - \gamma z)} + e^{-ik(\alpha y + \gamma z)} + \int\limits_{-\infty}^{\infty} a(\xi) e^{-ik(\sqrt{1-\xi^2} z + \xi y)}\, d\xi, \\ (z > 0) \\ \sum\limits_{m=0}^{\infty} [A_m e^{-ih_m (z+a_1)} + B_m e^{ih_m z}] \cos \dfrac{\pi m u}{2b}, \\ (-a_1 < z < 0) \\ \sum\limits_{m=0}^{\infty} [C_m e^{-ig_m (z+a_2)} + D_m e^{ig_m (z+a_1)}] \cos \dfrac{\pi m u}{2b}, \\ (-a_2 < z < -a_1) \\ \sum\limits_{m=0}^{\infty} [E_m e^{-ih_m (z+a_3)} + F_m e^{ih_m (z+a_2)}] \cos \dfrac{\pi m u}{2b}, \\ (-a_3 < z < -a_2) \\ \int\limits_{-\infty}^{\infty} d(\xi) e^{ik(\sqrt{1-\xi^2}(z+a_3) - \xi y)}\, d\xi, \ (z < -a_3) \end{cases} \tag{1}$$

where $\alpha = \sin\theta$, $\gamma = \cos\theta$ (θ being the angle of incidence), $u = y + b$, $h_m = \sqrt{k^2 \varepsilon_{r1} - (\frac{\pi m}{2b})^2}$, $g_m = \sqrt{k^2 \varepsilon_{r2} - (\frac{\pi m}{2b})^2}$, $\varepsilon_{r1} < \varepsilon_{r2}$, and A_m, B_m, C_m, D_m , E_m and F_m are amplitudes of parallel-plate waveguide mode m in the various regions of the slot, related to reflection and transmission coefficients at the interfaces. Applying boundary condition of continuity of function H_x and $\frac{1}{\tilde{k}^2} \frac{\partial H_x}{\partial z}$ where $\tilde{k} = k\sqrt{\varepsilon_r}$ at the interfaces between the dielectrics within the slot ($|y| < b$) at $z = -a_1$ and $z = -a_2$, we can obtain A_m, C_m, D_m and F_m in terms of B_m and E_m:

$$A_m = \frac{e^{-ih_m (a_3 - a_2)}}{T_{1m} + T_{2m}} \left[E_m - T_{3m} B_m e^{ih_m (a_3 - a_2 - a_1)} \right]$$

$$F_m = B_m e^{-ih_m a_1} \frac{T_{1m}^2 - T_{2m}^2 + T_{3m}^2}{T_{1m} + T_{2m}} - E_m \frac{T_{3m} e^{-ih_m (a_3 - a_2)}}{T_{1m} + T_{2m}}$$

where $T_{1m} = \cos g_m(a_2 - a_1)$, $T_{2m} = \frac{i}{2}\left[t + \frac{1}{t}\right] \sin g_m(a_2 - a_1)$, $T_{3m} = \frac{i}{2}\left[t - \frac{1}{t}\right] \sin g_m(a_2 - a_1)$ and $t = \frac{g_m \varepsilon_{r1}}{h_m \varepsilon_{r2}}$. We next apply the boundary condition $\frac{\partial H_x}{\partial z} = 0$ at surface of the metallic shield and continuity of H_x and $\frac{1}{k^2}\frac{\partial H_x}{\partial z}$ at the slot surfaces. Defining $\tilde{x}^{\pm}(\zeta) = [a(\zeta) \pm d(\zeta)]\sqrt{1-\zeta^2}$, where ξ and ζ are Fourier transform variables with respect to y, we arrive at the integral equations:

$$\tilde{x}^{\pm}(\zeta) = \frac{4\alpha(kb)^2\zeta}{\pi\varepsilon} \sum_{m=0}^{\infty} \frac{h_m b \Gamma_{2m}^{\pm}}{\Gamma_{1m}^{\pm}(1+\delta_{0m})} G_m(\zeta) G_m(\alpha) + \frac{2(kb)^2\zeta}{\pi\varepsilon} \sum_{m=0}^{\infty} \frac{h_m b \Gamma_{2m}^{\pm}}{\Gamma_{1m}^{\pm}(1+\delta_{0m})} \times \times \int_{-\infty}^{\infty} \tilde{x}^{\pm}(\xi) \frac{\xi}{\sqrt{1-\xi^2}} G_m(\zeta) G_m(\xi)\, d\xi \tag{2}$$

where

$$G_m(\zeta) = \frac{\sin(kb\zeta - \frac{\pi m}{2})}{(kb\zeta)^2 - (\frac{\pi m}{2})^2}$$

$\Gamma_{1m}^{\pm} = \frac{e^{-ih_m(a_3-a_2+a_1)}}{T_1+T_2} \pm 1 \mp \frac{T_3 e^{-ih_m(a_3-a_2+a_1)}}{T_1+T_2}$, $\Gamma_{2m}^{\pm} = \frac{e^{-ih_m(a_3-a_2+a_1)}}{T_1+T_2} \mp 1 \mp \frac{T_3 e^{-ih_m(a_3-a_2+a_1)}}{T_1+T_2}$ and δ_{0m} is the Kronecker delta.

For normal incidence ($\theta = 0$), we assume that $kb\sqrt{\varepsilon_{r1}} < \pi$, which ensures that all higher-order parallel-plate waveguide modes besides the TEM ($m = 0$) are cutoff. Retaining only the $m = 0$ terms in (2), we get a degenerate-kernel integral equation whose solution can be found by well-known methods:

$$\tilde{x}^{\pm}(\zeta) = \frac{2kb}{\pi\sqrt{\varepsilon_{r1}}} \operatorname{sinc}(kb\zeta) N^{\pm} \tag{3}$$

where: $N^{\pm} = \left[\frac{\left(\frac{\Gamma_{20}^{\pm}}{\Gamma_{10}^{\pm}}\right)}{1 - \frac{kb}{2\sqrt{\varepsilon_{r1}}}\left(\frac{\Gamma_{20}^{\pm}}{\Gamma_{10}^{\pm}}\right) I_{00}}\right]$, $I_{00} \simeq 2 - \frac{4i}{\pi} \ln\frac{\phi}{kb}$ for $kb \ll 1$ [1] and $\ln\phi = \frac{3}{2} - \gamma_E$ with $\gamma_E \simeq 0.5772\ldots$ being Euler's constant. Now we can determine the magnitude of the transmission through the infinite dielectric slot. Following the method of [7], we define the (amplitude) transmission factor as:

$$TF = \frac{\int_{-b}^{b} E_y(y, -a)\, dy}{2b|E_y(y, 0)|} = \frac{1}{\sqrt{\varepsilon_{r1}}}(N^+ - N^-) \tag{4}$$

having used (3) to obtain the final formula.

III. Results

As perhaps the simplest example, we assume that the gap is symmetrically located ($a_3 - a_2 = a_1$), and take a shield thickness $a_3 = 4$ mm, a slot width $2b = 2$ and $a_2 = 2.3$ mm, with $\varepsilon_{r1} = 50$ and $\varepsilon_{r2} = 1$ (air). A comparison among the result of equation(4), the analytical result for the case of no gap [7], and a full-wave finite-element simulation using Ansys HFSS is shown in Figure 2. The results show that introducing a gap within the slot will shift the even-order resonances to higher frequencies while keeping odd-order resonances almost unchanged. Increasing the size of the gap will shift the even-order resonances to progressively higher frequencies until they meet the next odd-order resonances, which will be shifted to higher frequencies but at a rate much slower than for the even-order resonances, eventually forming new resonances which will keep moving to higher frequencies as the size of the gap is increased. Because the gap is introduced in the middle of the slot in our example, the field of the odd-order resonances is almost zero at that location, but is at a maximum for even-order resonances, which explains why the impact of the gap on the latter was larger.

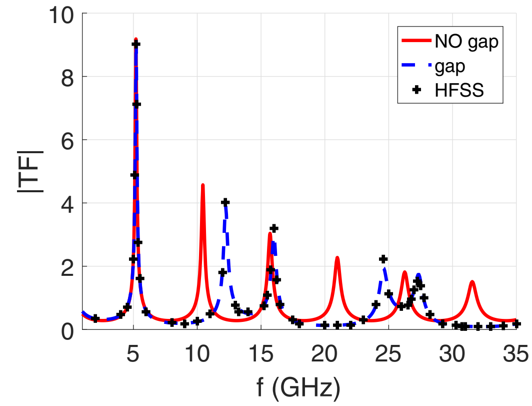

Fig. 2. Transmission Factor TF of homogeneous and inhomogeneous formula, $a_3 = 4$ mm, $b = 1$ mm and gap=0.6 mm

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