Analytical Effective Length Comparisons of Circularly Distributed Antenna Arrays

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***Abstract*—This work investigates a far-field quantity, known as the effective length, of one, two and three-dimensional arrays bound within circular constraints and its relation to the Fraunhofer region. Maximum gain is achieved for a volumetric array only when compared using the same effective length of linear and planar arrays.**

***Index Terms*—Antenna array and distributed beamforming.**

I. INTRODUCTION

A. *Expected value radiation pattern*

The potential function in the far-field regime of an isotropic point source is denoted as $\Xi(r_n, \psi_n)$:

$$\nabla^2 \Xi(R_n) + k^2 \Xi(R_n) = \delta(\vec{r} - \vec{r}_n) \tag{1}$$

This potential function meets the Fraunhofer condition ($r \geq 2D^2/\lambda$) and is independent of polarization as it behaves isotropically in nature. Hence, the isotropic potential does not need to be multiplied by either of the constitutive parameters ε or μ as provided by the common electric and magnetic vector potential [1-3]. For a finite sized radiator, this is written as (2) where I_n is its amplitude excitation [1]:

$$\begin{aligned}
&\nabla^2 G(R_n) + k^2 G(R_n) == \sum_{n=1}^{N} \frac{\delta(r-r_n)\delta(\theta-\theta_n)\delta(\phi-\phi_n)}{r r_n \sin\theta} \\
&G(R_n) = \underbrace{\sum_{n=1}^{N} I_n \frac{e^{-jk|r-r_n|}}{|r-r_n|}\delta(r-r_n)}_{Exact\ Solution} = g(r) \underset{Effective\ Length}{\ell} \qquad .(2) \\
&g(r) = \frac{e^{jkr}}{4\pi r}, \underset{Effective\ Length}{\ell} = \sum_{n=1}^{N} I_n \frac{e^{-jk|r-r_n|}}{|r-r_n|}\delta(r-r_n) = \int_{-\ell/2}^{\ell/2} I_n dr_n
\end{aligned}$$

The effective length relation of an antenna, whether linear or aperture, is a quantity to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it. This vector quantity is represented by $\vec{\ell}_{eff}(\theta,\phi) = \ell_\theta(\theta,\phi)\hat{\theta} + \ell_\phi(\theta,\phi)\hat{\phi}$. In the near-field the radiated wave is spherical in nature such that the effective length of the antenna is uniform throughout. However, the far-field approximation is such that the radial term of the Green's function $G_r \approx 0$. Hence, a finite dipole aligned along the z-axis has the components, $G_\theta = -\sin\theta$, $G_r = G_\phi = 0$ with an effective field angle $|G| = \sin\theta = \sin\psi_n$.

The total field of continuously distributed sources is simply the product of the element and space factors. The space factor is broken down into the product of its effective field angle and effective length (ℓ_{eff}). This is analogous to the pattern multiplication of distributed or discrete-element antenna arrays.

$$\begin{aligned}
&G(R_n) = g(r) \underbrace{\sum_{n=1}^{N} \sin\psi_n I_n e^{jkr_n \cos\psi_n}}_{Far-Field} = g(r) \underset{Effective\ Length}{\ell} \\
&\Xi(r_n, \psi_n) = \underset{Element\ Fcactor}{g(r)} \underset{Effective\ Field\ Angle}{\sin\psi} \underset{Effective\ Length}{\ell} \qquad (3) \\
&\underset{Effective\ Field\ Angle}{\sin\psi_n} \overset{Exact\ Solution}{=} \sqrt{\hat{r}_n\cdot\hat{r}_n + \hat{\theta}_n\cdot\hat{\theta}_n + \hat{\phi}_n\cdot\hat{\phi}_n} = 1, \\
&\overset{Far-Field}{\approx} \left|\sqrt{1-\cos\psi_n^2}\right| = \sqrt{1-\hat{r}_n\cdot\hat{r}_n} = \sqrt{\hat{\theta}_n\cdot\hat{\theta}_n + \hat{\phi}_n\cdot\hat{\phi}_n}
\end{aligned}$$

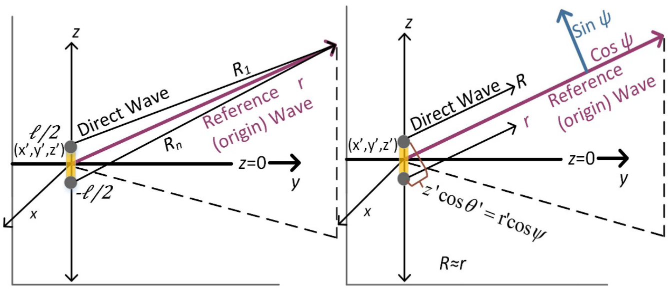

Fig. 1. Small dipole with exact solution of radiated fields (left). Far field approximation (right).

B. *Effective length calculation of a small dipole* (λ << *50*)

A small dipole with uniform current excitation, co-polarization, and neglible polarization disparity throughout its length oriented along the z-axis takes the form:

$$\begin{aligned}
&total\ field = (element\ factor)\ x\ (space\ factor) \\
&\left\{ \underset{Element\ Fcactor}{g(r)} \left| \underset{Space\ Factor}{\sum_{n=1}^{N} I_n \sin\psi_n e^{jkr_n\cos\psi_n}} \right. \right\} \qquad (4) \\
&= g(r)\sin\psi \int_{-\ell/2}^{\ell/2} dz' = \underset{Element\ Fcactor}{g(r)} \underset{Effective\ Field\ Angle}{\sin\psi} \underset{Effective\ Length}{\ell} \\
&\neg I_n = 1 \forall n, \sin\psi = \sqrt{(-\sin\theta)^2 + 0^2} = \sin\theta
\end{aligned}$$

C. *Effective length comparisons*

The volume, area, and effective length of the first four characteristic modes of the Sinc family (n = *0-4*) [3-4]; spherical distributed array (*SDA*), circular distributed array (*CDA*) [5], linear distributed array (*LDA*), and ring distributed array (*RDA*) are provided in (6); note that not all configurations are applicable.

$$\begin{array}{ccccc} & SDA & CDA & LDA & RDA \\ Volume & 4\pi A^3/3\lambda^3 & NA & NA & NA \\ Area & 4\pi A^2/\lambda^3 & \pi A^2/\lambda^2 & NA & NA \\ Length(\ell_{eff}) & 8\pi A/\lambda^3 & 2\pi A/\lambda^2 & 2A/\lambda & 2\pi A/\lambda \end{array} \quad (5)$$

An *SDA* has an effective length approximately twelve times greater than the *LDA* and four times that of either *RDA* or *CDA* (provided λ=1). Solving for the effective aperture radius A allows a fair comparison of effective length such that:

$$\begin{array}{ccccc} & SDA & CDA & LDA & RDA \\ A= & \lambda^3/8\pi & \lambda^2/2\pi & \lambda/2 & \lambda/2\pi \end{array}. \quad (6)$$

An evaluation of the effective length of a SDA, CDA, LDA and RDA is given using two different methods. The first method bounds all topologies to the same aperture size (A=1). The second assessment compares the effective length such that ℓ_{eff} = 10 π for all topologies. Results of each are provided respectively in Figs. 2 and 3. Numerical calculations (Tables 1 and 2) show a comparison on the performance of each array type. Maximum gain is achieved for a volumetric array when ℓ_{eff} is equal for all arrays. However, when all aperture sizes (A) are set equal, the RDA performs best.

II. Isotropic Radiator Directivity in Angular Space

The total uniform and normalized far-field isotropic components Ξ (r_n, ψ_n) emit a radiation intensity that is predominately real (depends on η) in its far zone region as provided by derivation of (8):

$$D_0(\theta_0,\phi_0) = 4\pi U(\theta_0,\phi_0)/P_{Rad} = 1,$$
$$U(\theta_0,\phi_0) = \left(\frac{1}{2\lambda}\right)^2 \left[\frac{1+\eta(\theta_0,\phi_0)^2}{\eta(\theta_0,\phi_0)}\right], P_{Rad} = 4\pi U(\theta_0,\phi_0) . \quad (7)$$

Hence, as long as η is independent of angle, the directivity is uniform and equal to unity in all angular spaces for a single isotropic radiator. This grows to a value of N for a collection of isotropic radiators such that

$$D = 4\pi A_{eff}/\lambda^2 = N$$
$$\Rightarrow A_{eff} = N\left(\lambda^2/4\pi\right), \text{Gain of an isotropic radiator} \quad (8)$$

where A_{eff} is the commonly accepted notation of the accepted effective area.

Table 1. A = 1 and λ=1.

	SDA	CDA	LDA	RDA
D_0	17.15dB	21.95dB	16.26dB	22.37dB
θ_{3dB}	25.96°	36.45°	21.97°	30.46°
ϕ_{3dB}	26.46°	5.49°	22.47°	3.5°
ASL $\theta_{cut\ plane}$	-29.30dB	-28.35dB	-18.59dB	-16.15dB
ASL $\phi_{cut\ plane}$	-23.26dB	-29.37dB	-18.57dB	-26.97dB

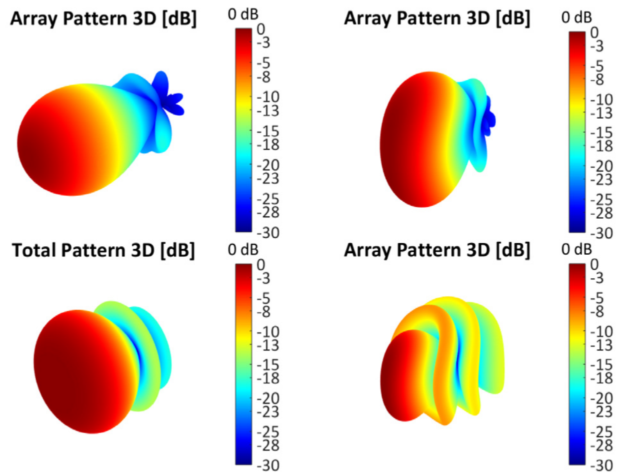

Fig 2. A=1and λ=1. SDA-top left; CDA-top right; LDA-bottom left; RDA-bottom right.

Table 2. Length = 10 π, (5) for all topologies.

	SDA	CDA	LDA	RDA
D_0	15.28dB	12.24dB	6.62dB	13.63dB
θ_{3dB}	32.95°	83.38°	90.87°	69.40°
ϕ_{3dB}	32.45°	29.46°	91.37°	20.47°
ASL $\theta_{cut\ plane}$	-29.25dB	-22.10dB	-8.62dB	-11.19dB
ASL $\phi_{cut\ plane}$	-29.13dB	-28.04dB	-8.62dB	-23.12dB

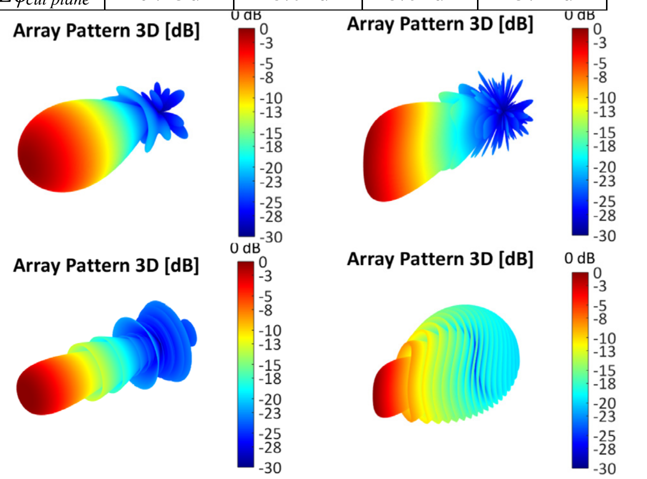

Fig 3. The aperture size A is solved independently for each topology in (5) such that ℓ_{eff} = 10 π and λ =1. SDA-top left; CDA-top right; LDA-bottom left; RDA-bottom right.

References

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