Umbrella Reflector Characterization for CubeSats: Analytical Formulation for Boresight Gain Loss

Vignesh Manohar and Yahya Rahmat-Samii
Department of Electrical and Computer Engineering
Univeristy of California, Los Angeles
California, USA
vignesh04@ucla.edu, rahmat@ee.ucla.edu

***Abstract*—As the needs for high gain antennas for CubeSats begin to evolve, deployable reflector antennas have re-gained significant interest. A particular class of deployable reflectors known as umbrella reflectors have been considered for several CubeSat missions. The umbrella reflector surface consists of a discrete number of parabolic ribs that are connected through surfaces called gores. The gores cause the surface to deviate from that of an ideal paraboloid causing phase deviations in the aperture, ultimately leading to reduced gain. The choice of the number of ribs is a critical design consideration for CubeSat antenna designs as it provides the balance between mechanical complexity and RF loss. This work extends the previous works to develop closed form expressions that relate the gain loss to the number of gores for a given diameter, rib focal length and frequency. The results from closed form expressions are compared with PO simulations to verify our formulations.**

I. INTRODUCTION

As the desire to explore space further with CubeSats evolves, it has become essential to develop high gain antennas that can be integrated with the small CubeSat form factor [1]. A potential candidate for such high gain antennas is the umbrella reflector, that can be stowed during launch and deployed once in space [2]–[4] as shown in Fig. 1.

The surface of an umbrella reflector consists of a finite number of parabolic ribs, which are connected through surfaces called gores. Each gore surface represents a part of a parabolic cylinder, bound between parabolic ribs. In many deployable systems, the gore surface is formed by stretching a mesh between the ribs. The gores cause the surface of the reflector to deviate from a true paraboloid resulting in a non zero RMS surface error between the umbrella reflector and the ideal paraboloid, causing reduction in gain. Thus, the choice of the number of ribs is a balance between the mechanical complexity and antenna gain loss. This tradeoff is especially critical for CubeSats owing to the limited space within the satellite chassis.

Several works have characterized the performance of the umbrella reflector. Of particular interest in this work is the best-fit approach of characterizing umbrella reflectors which was introduced in [5] and extended in [6]. This work further extends this concept to develop simple closed form expressions relating the gain loss to the parameters of the umbrella reflector for a given frequency. Comparisons with PO simulations are also provided to demonstrate the validity of the closed form expressions.

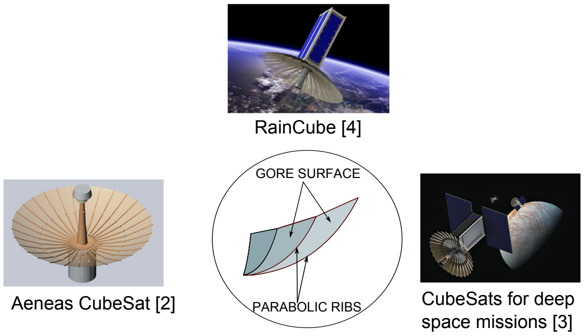


Fig. 1. Representative CubeSat missions employing umbrella reflectors to provide high data rates and resolution.

II. MATHEMATICAL REPRESENTATION OF THE GORE SURFACE

For a reflector consisting of N_g gores, with a rib focal length of F_r, the equation of the gore surface can be expressed as [7]:

$$z_g = \frac{\rho'^2}{4F_g(\phi')},\ F_g(\phi') = \frac{F_r \cos^2\left(\pi/N_g\right)}{\cos^2\frac{\phi_m+\phi_{m+1}-2\phi'}{2}} \qquad (1)$$

where, $\phi_m = \frac{2\pi}{N_g}(m-1)$ for $m = 1, 2, ..N_g$. The variables ρ' and ϕ' are cylindrical coordinates as defined in Fig. 2. The focal length of the umbrella reflector is a periodic function of the azimuthal angle ϕ' as can be seen from (1). The absence of a distinct focus makes the optimal feed position (or the subreflector position for dual reflector systems) ambiguous. Different approaches to estimate the optimal feed position were reviewed in [6].

III. ANALYTICAL FORMULATION OF GAIN LOSS

This section extends the best-fit paraboloid approach presented in [5] to develop closed form expressions for the boresight gain loss. We also include the effect of amplitude taper in our calculations. Assuming that the aperture taper can be modelled by a 2-parameter taper distribution [8], the following expression for the weighted RMS error can be developed [9]:

$$\Delta z_{rms} = \sqrt{\frac{1}{A_g}\int_0^{\frac{D}{2}\cos\phi_\circ}\int_0^{y'\tan\phi_\circ}(z_g - z_{eq})^2 Q(\rho')dx'dy'} \qquad (2)$$

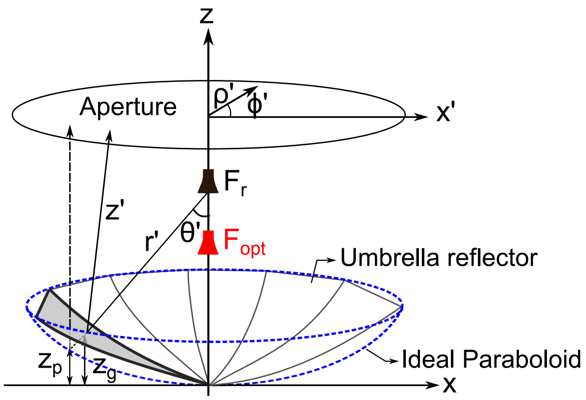
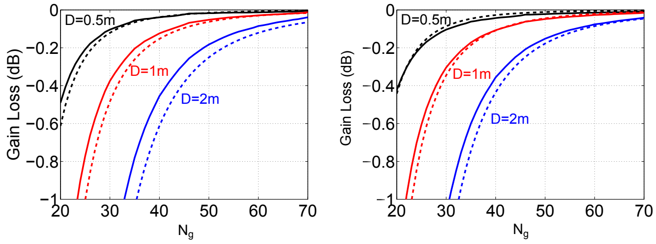


Fig. 2. Diagrammatic representation of various parameters and coordinate systems used for analyzing the umbrella reflector.



(a) (b)

Fig. 3. Gore loss as a function of number of gores when the feed is kept at the optimal feed location for different aperture diameters D and $F_r/D = 0.5$ at Ka band (35.75 GHz). (a) 0 dB feed taper and (b) 10 dB feed taper. The solid lines show the simulated gain loss using PO and the dotted lines show the gain loss computed by (6) for 0 dB amplitude taper (and numerically solving (2) to get RMS error for 10 dB taper).

Where $Q(\rho')$ and A_g are the amplitude taper function and the effective area, respectively, given by

$$Q(\rho') = C + (1 - C)\left[1 - \left(\frac{\rho'}{D/2}\right)^2\right]^p \tag{3}$$

$$A_g = \int_0^{\frac{D}{2}\cos\phi_\circ} \int_0^{y'\tan\phi_\circ} Q(\rho')dx'dy' \tag{4}$$

where the edge taper is given as $ET = 20\log_{10} C$ and $\phi_0 = \pi/N_g$. The value of p in (3) governs the slope of the taper, typically chosen to be 1.6.

For a uniform taper distribution ($C = 1$), the equation for the RMS error in (2) can be analytically evaluated to give a closed form expression for the RMS error [5]. For practical scenarios where the number of gores are reasonable ($N_g > 20$), a simplified equation for the RMS error can be developed [5]:

$$\Delta z_{rms} = 0.010758\frac{D}{F_r/D}\left(\frac{\pi}{N_g}\right)^2 \tag{5}$$

Substituting this into the Ruze's equation [8] yields

$$\Delta G(dB) = 685.811\left(\zeta\frac{\Delta z_{rms}}{\lambda}\right)^2$$

$$= 7.73\frac{\zeta^2}{\lambda^2}\frac{1}{(F_r/D)^2}\left(\frac{\sqrt{D}}{N_g}\right)^4 \tag{6}$$

Where $\zeta = (4F_r/D)\sqrt{\ln[1 + 1/(4F_r/D)^2]}$. This leads to a very interesting result: for a given F_r/D, the gain loss scales as the $(\sqrt{D}/N_g)^4$. Thus, if the diameter is doubled, the number of gores must only increase by a factor of $\sqrt{2}$ to maintain the same loss. Representative simulation results highlighting the dependency of gain loss on N_g for various aperture diameters at Ka band (35.75 GHz) are shown in Fig. 3. It should be noted that these results are valid only if the feed is kept at the optimal location, which can be estimated by $F_{opt} = F_r\left(1 - \frac{2}{3}\left(\frac{\pi}{N_g}\right)^2\right)$ when the gore loss is within 3dB [6].

IV. CONCLUSION

This paper revisited the analysis of umbrella reflectors with the intent of developing a relationship between the gain loss, parameters of the umbrella reflector (number of gores, aperture diameter and rib focal length) and frequency. Combining the RMS error from the best-fit approach and Ruze's equation, it was shown that the gain loss scales as $(\sqrt{D}/N_g)^4$ for a given F_r/D and frequency. The effects of amplitude taper were also incorporated into analysis. The validity of the closed form expressions was shown through comparisons with PO simulations.

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