Spherical FDTD Numerical Dispersion Analysis

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The numerical dispersion relationships (NDR) of finite difference time domain (FDTD) algorithms predict the effect of spatial and temporal discretizations on the numerical wave number. Therefore, it is useful in predicting phase errors in FDTD wave solutions. The NDR for Cartesian coordinates-based FDTD is obtained by substituting plane waves harmonics into the discretized Maxwell's equations. On the other hand, the NDR for Cylindrical FDTD is obtained by substituting cylindrical wave harmonics into the discretized Maxwell's equations. (M. F. Hadi et al., *IEEE Transactions on Antennas and Propagation*, 2017.)

The current study investigates the derivation of the NDR for Spherical FDTD. Spherical wave harmonics are used to obtain the dispersion relationship. In addition to dependence on spatial discretizations, the resulting NDR also shows dependence on absolute radial distance and elevation angle. The NDR is validated in the continuum limit where it converges to the free-space dispersion relationship $\beta^2 = \omega^2\mu\epsilon$. It is also validated in the far-field region where it converges to that of a one-dimensional Cartesian FDTD, as expected.

Spherical FDTD NDR is analyzed to study the sensitivity of wave number solutions to, in particular, absolute location within the spherical grid. The figure below shows the sensitivity of wave number to the radial distance for several grid resolutions represented by "R" and for rotational θ and ϕ mode numbers $n = 2$ and $m = 0$, respectively.