Seeing the Invisible: Imaging Hidden Features with Multiple-Scattering Reconstruction

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***Abstract*—Single-scattering formulations can reconstruct only a small portion of the object-contrast spectrum when the transmitters and receivers have a limited access of bistatic-angle sector. However, when multiple-scattering phenomena is involved, the measurements acquire richer spectral information, which can be exploited for reconstructing a larger spectral area. This paper demonstrates multiple-scattering reconstructions, extracting spatial features which are hidden to conventional formulations.**

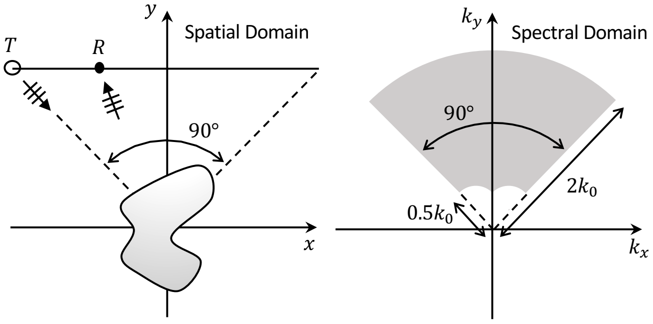
I. INTRODUCTION

Today's practical imaging methods usually omits the wave interactions such as diffraction, and multiple scattering. Diffraction tomography, however, includes quite wave phenomena with a single-scattering assumption, i.e., Born approximation, and provides a linear relationship between the scattering object-contrast and the scattered field. This is successfully implemented for many cases, where the object is illuminated by a set of transmitters from different angles and scattered field to all-around are collected by a set of receivers.

In several real-life applications including but not limited to geophysical, ultrasound, and sonar imaging as well as radar and microwave imaging, one may not have a luxury to place transmitters and receivers around the object but can place them into a limited angle sector [3]. In this case, one can only extract spatially varying edges in subject to direct specular reflection but cannot image slowly varying areas and edges of indirect reflections with conventional diffraction tomography. But, in fact, the scattered field involves richer information than what linear methods can extract, due to the multiple-scattering phenomena, especially when the object is highly scattering, i.e., when the incident wave internally bounces many times by the scatterer. This information can be extracted through the distorted-Born approximation [2], and incorporated into imaging with iterative nonlinear optimization methods, like the distorted-Born iterative method (DBIM) [4]. This paper is on limited-angle imaging of spatial features using the multiple-scattering information.

II. GEOMETRICAL AND PHYSICAL INTERPRETATION

Diffraction-tomography formulation provides a geometrical insight of the relation between the measurements and corresponding spectral information. Now, consider an object at the origin and a set of transmitters and receivers placed on a line with access to 90° bistatic angle sector, as showed in Fig. 1(a). Here, transmitters and receivers are modeled with Dirac-delta functions, and placed in the far-field region of the object. They fire a pulse with a frequency bandwidth of $\omega_0/4$–ω_0, where ω_0 is the highest frequency. A single transmitter fires at a time and all receivers collect scattered field at once from the limited bistatic angle sector. Under this scenario, the diffraction-tomography formulation maps the acquired measurement into the shaded area of the object spectrum as showed in Fig. 1(b).



(a) (b)

Fig. 1. (a) A limited-angle observation scenario where transmitters and receivers have a angle of sight of 90°. The transmitters/receivers operates between $\omega_0/4$–ω_0 bandwidth. (b) The spectral area that can be sampled with a single-scattering formulation is shaded with gray. The low-frequency components (including the DC component at the origin) is missing.

As evident from the geometrical argument above, the low-frequency components in the k_y axis and the high-frequency components in the k_x axis are not accessible. As a results, with a single-scattering mechanism, only less than a quarter of the area falls into the diffraction limit of $2k_0$ can be sampled by a transmitter/receiver pair, where k_0 is the wavenumber at the highest operating frequency. Consequently, a reconstruction method based on Fourier transform or linear least-squares cannot reveal the the corresponding spatial features of the object.

Consider the DC component, for example, which can be acquired at the forward-scattering direction with the first-order scattering. Physically, a plane wave passing through the dielectric object integrates the dielectric-permittivity contrast, which corresponds the DC component at the object-contrast spectrum. But in a limited-angle scenario, it is impossible to capture the DC component since one cannot place a receiver at the other end of transmitters. Therefore, the reconstruction is a high-pass filtered version of the object contrast where only edges involved in specular reflection are appeared.

When the second-order scattering is significantly involved, on top of the first-order scattering, there is an additional term comes from the corresponding the scattered wave going back to the receivers propagates through and integrates the dielectric permittivity contrast, which eventually carries the DC component. Higher-order scattering terms additively involve more information about the object spectrum, but difficult to analyze which term involves which information. As opposed to a single-scattering reconstruction, the least-squares problem with multiple-scattering is nonlinear, and therefore has to be solved iteratively with successive linearizations, as in DBIM.

III. NUMERICAL RECONSTRUCTIONS

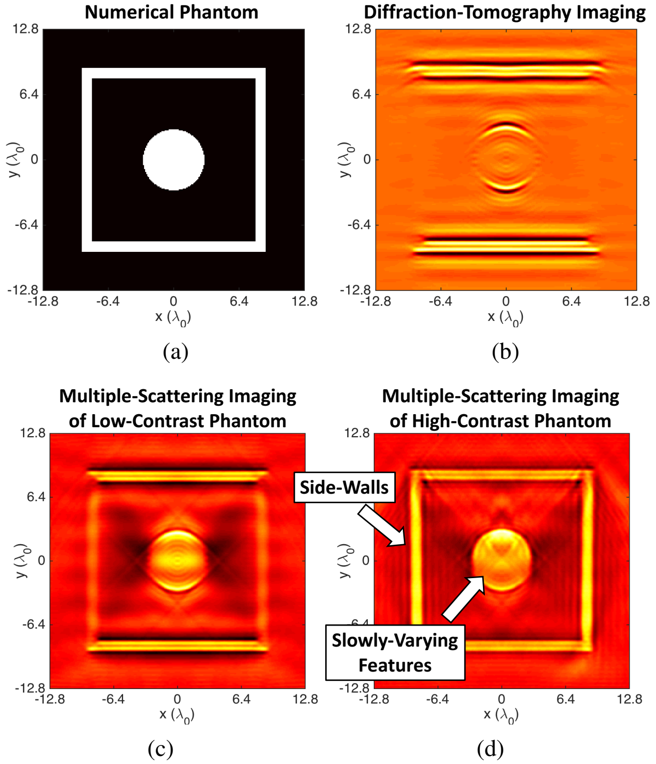
For demonstrating the multiple-scattering imaging with limited angle of observation, a two dimensional numerical setup is designed as described in the previous section. A phantom shown in Fig. 2(a) is placed at the origin and illuminated by 32 transmitters and the scattered field is collected by 128 receivers, where all placed inside inside a 90° bistatic angle sector. A DBIM solver is implemented for the inverse multiple-scattering problems involving a square imaging domain with $12.8\lambda_0$ side-length, where λ_0 is the free-space wavelength at the highest frequency, ω_0. The imaging domain uniformly discretized with $0.1\lambda_0$-size pixels. 128 frequency points are for solving the time-domain problem with frequency-domain solver. The scattered field is collected via full-wave numerical solutions of the forward problems.

Figure 2 shows three reconstructions. The first one in Fig. 2(b) shows reconstruction with diffraction tomography formulation, which assume single scattering via Born approximation. Fig. 2(c) shows the multiple-scattering reconstruction involving the phantom with the contrast of 0.01, where the higher-order scattering terms are weak. As it is seen, the slowly-varying features and traces of the side-wall appears. In the last reconstruction, shown in Fig. 2(d), the scattering contrast of the phantom is 0.1, where the higher-order terms are significant, and consequently, the problem is highly nonlinear. The last reconstruction shows that the internal multiple-scattering enriches the scattered field, and consequently the nonlinear reconstruction can extract the side-walls and slowly-varying features apparently.

The reconstructions are obtained with nonlinear conjugate-gradient method with Polak-Ribiére steps. The step size is found approximately by fitting a second-order curve with an additional forward solution, i.e., step is taken on the bottom of the one dimensional curve. Reconstructions use no regularization, except the early termination (all solutions are terminated heuristically when the solution converges). Evident from the trend in the reconstructions, the nonlinear optimization boils down to a linear one when the object contrast is low because the cost functional is almost quadratic when the contrast is low and higher-order scattering terms are weak.

IV. CONCLUSION

A single-scattering mechanism can reconstruct only a small portion of the object-contrast spectrum when transmitters and receivers are placed inside a limited angle sector. This spectrum spatially corresponds to the edges which are involved in a specular reflection. When object is in subject to significant higher-order scattering phenomena, the scattered the multiple-scattering information in the scattered field can be exploited for reconstructing the missing spectral components corresponding slowly-varying ares and edges which are not involved to a direct specular reflection. This paper demonstrates multiple-scattering solutions for imaging such features.



(a) (b)

(c) (d)

Fig. 2. (a) Shape of the numerical phantom. (b) Diffraction-tomography imaging of the phantom with scattering contrast of 0.1. (c) Multiple-scattering imaging of the phantom with scattering contrast of 0.01. (d) Multiple-scattering imaging of the phantom with scattering contrast of 0.1. When multiple-scattering is significant, side-walls and slowly-varying features of the numerical phantom appears in the nonlinear reconstruction.

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