## EM Simulation and Characterization of Underground Mines Using Ray Tracing, Vector Parabolic Equation, and Hybrid Approaches

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This paper addresses application of computational electromagnetics (CEM) to signal propagation modeling in underground mines. One of our main approaches to the wireless propagation analysis of underground mines, which is an extremely challenging CEM problem, relies primarily on shooting-bouncing rays (SBR) ray-tracing. For uniform or nearly uniform tunnel sections, we also use vector parabolic equation (VPE) modeling, while image-theoretic ray-tracing (IT) is used to compute phase information and exact paths as an augmentation to the SBR approach. While previous work has been performed, with the associated results presented, on the application of hybridized ray-tracing/VPE methods to wireless signal characterization of mostly uniform structures such as railway tunnels, little work has been done toward applying and optimizing these methods to work with more-complex geometries such as those present in mine tunnels.

We discuss the computational complexity and accuracy tradeoffs of these approaches and the associated benefits of hybridization toward generally more-robust, faster solvers. We also present results for test structures including ideal-waveguides, lossy waveguides, smooth-walled tunnels, rough-walled tunnels, and mine tunnel models from real-world LiDAR data. This forms a basis for discussion on which of these sub-techniques are bestsuited to which geometries given practical computational and data constraints. Hybridization with full wave method-of-moments surface integral equation (MoM-SIE) and finite-element method (FEM) approaches is discussed as well.

We also examine the degree of complexity required to accurately model non-uniform tunnels and the corresponding degree to which these models can be simplified toward faster computation. The associated error introduced by model simplification is examined in comparison to the associated speed-up of the solver, and heuristics are discussed for making geometric simplifications. VPE solvers, benefiting from models of uniform crosssection are used for long, straight tunnel segments where the error in the wall diameters from a uniform cross-sectional profile is below a desirable threshold. We introduce meshing methods for generating VPE meshes of uniform cross-section that can be interfaced with ray-tracing-based models.

The trade-offs between the flexible but computationally-expensive ray-tracing methods and the fast but rigid VPE method are discussed with regards to the generation of volumetric field data. Furthermore, we examine the practical challenges associated with collecting and processing geometric data in rough-walled tunnel environments and introduce approaches to minimize error. The methods by which we construct surface meshes and volume meshes from these data are discussed in brief, while we put more emphasis on the practical considerations and simplifications that can and cannot be made. We leverage several techniques from the computer-graphics community toward our application and discuss the efficiency of the associated algorithms.