Shaped-Profiled and Material-Engineered Inhomogeneous Lens Antennas: GO Curved Ray Tracing and Aperture Fields

Jordan Budhu* and Yahya Rahmat-Samii
Electrical Engineering
UCLA
Los Angeles, California, USA
jordan.budhu@ucla.edu, rahmat@ee.ucla.edu

***Abstract*—In this paper, we present an algorithm for tracing rays through an isotropic inhomogeneous medium with the aim to incorporate the algorithm in a lens synthesis technique based on Geometrical Optics and Particle Swarm Optimization (PSO). The algorithm tracks the phase, amplitude, and polarization variation of the electric field along the ray, which in this case becomes a space curve. The rays are found by solving the light-ray equation numerically using the Runga-Kutta to the 4th order (RK4) algorithm. Key governing equations are summarized allowing the implementation of numerical techniques for the complete determination of GO fields. Validation of the ray paths are given by comparing the numerical results with the analytic formulation of the ray paths in an ideal Luneburg Lens Antenna. The results show good agreement.**

I. INTRODUCTION

Shaping dielectric lens antenna via an optimization based synthesis technique requires an efficient numerical program to analyze the lens performance for a given geometry and permittivity profile. The asymptotic technique of Geometrical Optics is a viable candidate. The efficiency of the method allows one to integrate it within a synthesis loop involving an optimization routine such as PSO. This technique makes use of the concepts of differential geometry to track the phase, amplitude, and polarization of the electromagnetic field viewed in the framework of rays (space curves) and wavefronts (surfaces). Two equations result from the asymptotic evaluation of the wave equation; the Eikonal Equation which can be solved to obtain the ray parameterization, and the Transport Equation, which can be solved to obtain the variation of the electric field along the ray. The key equations allowing numerical solutions are summarized in the paper by Yeh [4]. These results are implemented in a GO/PSO synthesis program to design shaped inhomogeneous lenses for spinning conically scanned beams. The validation of the program is an important step. Here we present validation for the ray paths by comparing the results with analytic results for the ideal Luneburg Lens [1]. Optimizations of both the surface expansion coefficients and the material permittivity coefficients for lenses is planned. This work is largely a continuation of the work presented in [1] except for the incorporation of capabilities of optimizing the lens inhomogeneity.

II. GEOM. OPTICS FOR INHOMOGENEUOS MEDIA THEORY

Fig. 1 shows a pictorial view of the procedure to trace rays to the aperture through an inhomogeneous lens. To obtain the aperture fields, the ray paths through the inhomogeneous lens must be

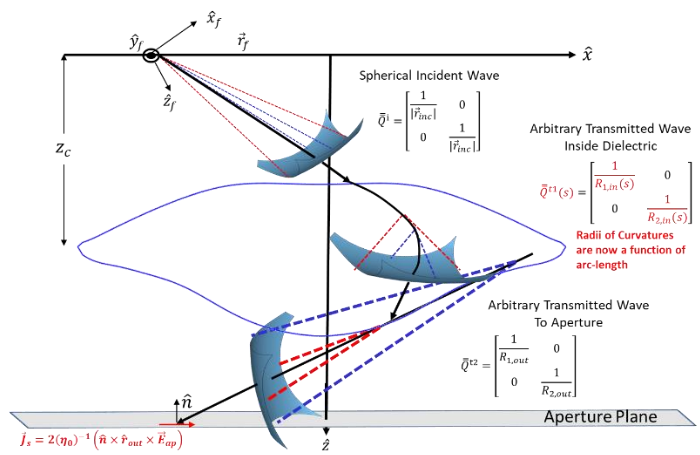

Fig. 1. Schematic view of ray tracing through inhomogeneous lens to obtain aperture fields

obtained. Here we summarize the key formulas necessary to determine the traced rays from the feed, through the inhomogeneous lens and to the aperture.

A. Eikonal Equation Solution for Ray Path

The Eikonal Equation can be recast into the light-ray equation

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)=\nabla n \qquad (1)$$

where n is the index of refraction, $\vec{r}$ is the position vector of the ray, and s is the arc-length parameter. (2) can then be cast into the First Order ODE [2]

$$\frac{d\vec{T}}{dt}=n\nabla n \qquad (2)$$

where $ds = ndt$ and $\vec{T}$ is the ray tangent. (3) equation above can be solved by the Runga-Kutta method (RK4) [2]. RK4 produces a series of points and tangent vectors along the ray.

These are then interpolated using Cubic Spline Interpolation to obtain the full parameterization of the ray. The parameterized ray can then be integrated to obtain the Optical Path Length of the ray between two points on the ray A and B. Note, S is a function which defines the optical path length between some reference point and a point on the ray.

$$S(B) - S(A) = \int_A^B n^2 dt \quad (3)$$

The numerical results of (3) are given in Section III for the ideal Luneburg Lens.

B. Transport Equation for Fields

The Transport Equation is

$$\frac{\partial \vec{e}}{\partial s} + \frac{1}{2}\left(\frac{1}{n}\nabla^2 S - \frac{\partial \ln \mu}{\partial s}\right)\vec{e} + \left(\vec{e} \bullet \nabla \ln n\right)\hat{t} = 0 \quad (4)$$

where $\vec{e}$ is the electric field vector, μ is the permeability of the medium and $\hat{t}$ is the unit ray tangent. (5) is solved in two parts, first for the amplitude variation and second for the variation of the direction [3]. To solve (5) for the amplitude, one multiples through by $\vec{e}^*$ to form the magnitude of $\vec{e}$. After some algebraic manipulation, one obtains

$$\left(\frac{|\vec{e}|}{\sqrt{\mu}}\right)_B = \left(\frac{|\vec{e}|}{\sqrt{\mu}}\right)_A \left(\frac{n(A)}{n(B)}\right)^{1/2} e^{-\frac{1}{2}\int_A^B (\nabla \bullet t) n dt} \quad (5)$$

(6) can be used to follow the amplitude variation along the ray. Note, it is well known theorem in differential geometry that the $\nabla \cdot \hat{t}$ can be related to the mean curvature of the wavefront surface, namely

$$\nabla \cdot \hat{t} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 2\kappa_m \quad (6)$$

(6) can be modified to include this definition. Next, (5) is again solved for the variation of the direction of the electric field along the ray by dividing through by $\sqrt{\vec{e} \cdot \vec{e}^*}$ to form an equation describing the unit vector $\hat{e}$

$$\frac{\partial \hat{e}}{\partial s} = -\left(\hat{e} \bullet \nabla \ln n\right)\hat{t} \quad (7)$$

(7) can be used to follow the polarization of the electric field along the ray.

III. Results

Results of RK4 numerical solution to (3) for the ideal Luneburg Lens is shown in Fig. 2a (in solid blue). The RK4 method was chosen for its algorithmic simplicity and accuracy. Overlaid on the same figure are the analytic results for the rays of an ideal Luneburg Lens (in dashed red) as given by the parametric equation [1]

$$\rho^2 = \frac{\sin^2 \delta}{1 - \cos\delta \cos(2\theta - \delta)} \quad (8)$$

where ρ and θ are the polar variables of the position vector of the ray defined from the center of the lens to a point on the ray and δ is the transmission angle of the ray measured from the lens axis to the tangent to the initial segment of the ray launched from the lens focal point as shown in Figure 2.

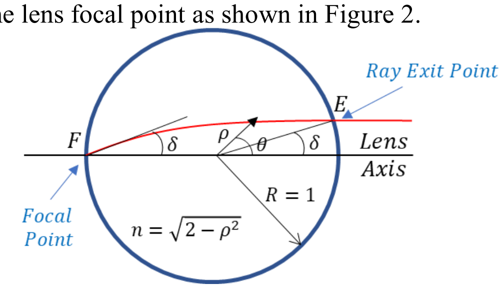

Fig. 2. Definition of parameters describing ray trajectories in analytic formulation of (8)

The results of Fig. 3a indicate the ray tracing algorithm of the developed code is working as expected. This code will then incorporate the results of Section II to follow the amplitude and polarization as part of a numerical synthesis code for shaped engineered material lenses as a follow-up to the work presented in [1]. For a simple verification of the entire program, a uniform amplitude electric field was forced in the projected aperture while keeping the true phase. The far field patterns are shown in Fig. 3b.

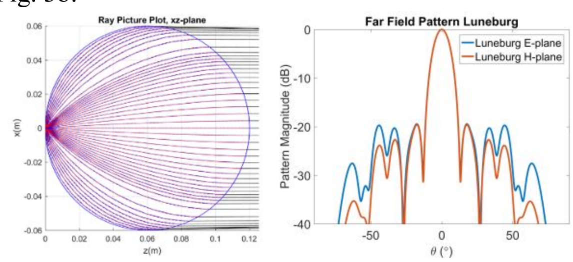

Fig. 3. (a) *left:* Rays Traced by RK4 numerical technique (solid blue) and by analytic expression of (8) (dashed red). (b) *right:* Far field patterns with forced uniform amplitude in projected aperture.

References

[1] Budhu, J., Rahmat-Samii, Y., "Synthesis of 3D-Printed Dielectric Lens Antennas Via Optimization of Geometrical Optics Ray Tracing," IEEE APS Conference, San Diego, CA, 2013.

[2] Schrank, H.E., "Graphical Construction of Rays in an Ideal Luneburg Lens," IRE Trans. On Ant. and Prop., pp. 410-411, Februrary, 1961.

[3] Sharma, A., Kumar, D. V., Ghatak, A.K., "Tracing Rays through Graded-Index Media: A New Method," Applied Optics, Vol. 21, No. 6, pp. 984-987, March 1982.

[4] Yeh, K.C., Lui, C.H., Theory of Ionospheric Waves, Chap. 5, Academic Press, New York, NY, 1972.

[5] Kline, M., "A Note on the Expansion Coefficient of Geometrical Optics," Comm. On Pure and Appl. Math, Vol XIV, pp. 473-479, 1961.