Using complex frequency-plane branch points to identify exceptional points of degeneracy in parity-time symmetric systems

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We consider the connection between critical point (CP) theory and exceptional points of degeneracy (EPD) in parity-time (PT) systems with loss and gain. EPDs occur when both system eigenvalues and eigenvectors coalesce for some values of system parameters (frequency, loss/gain values, etc.). Such degeneracies lead to non-hermitian physics and to interesting physical phenomena, such as unidirectional reflection/transmission systems, chiral and unidirectional emission, unidirectional invisibility, and a host of related effects.

Alternately, in previous work on guided-wave structures we have shown that different modal interaction phenomena are related to CPs of dispersion function, which in turn correspond to complex frequency-plane branch points (CFPBP) leading to modal degeneracies and mode exchange. In this work, we connect our previous theory of CP-CFPBPs and EPDs, showing that the same equations which model CP-CFPBPs also govern EPDs for the case of second-order degeneracies. We apply our results to a PT-symmetric coupled transmission line system recently considered in the literature, and we consider a generalized case where EPDs occur in complex-frequency space. We show that transmission line systems inherently possess EPDs (simultaneous eigenvalue and eigenvector degeneracies), and how system parameters R,L,C affect EPD locations. We demonstrate that for the case of real valued EPDs a characteristic dispersion behavior in the neighborhood of EPDs corresponds to the fold bifurcation of the dispersion function. Based on the CP-CFPBP formalism, closed-form expressions for EPDs frequency and wavenumber have been derived. We also consider a system consisting of two 2D parallel plates in a PT-symmetric situation, which also admits EPD/CP-CFPBPs.