Simulation of Electron Bernstein Waves by Charge-Conserving EMPIC on Irregular Meshes

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***Abstract*—We investigate waves propagating, and polarized perpendicular to the stationary magnetic field, in a magnetized warm plasma by means of electromagnetic particle-in-cell (EMPIC) simulations. Conventional EMPIC algorithms on irregular grids violate charge-conservation because the ensuing charge continuity equation leaves residuals at the discrete level. It is shown that the use of non-charge-conserving scatter algorithms in EMPIC simulations may alter the physics due to the presence of a spurious static field produced by remnant space charges and, as a result, produce noisy spectral bands. A newly developed charge-conserving EMPIC solver is shown to produce spectral bands exhibiting less noise.**

I. INTRODUCTION

Electron Bernstein wave (EBW) [1], [2] has received a great deal of attention since it is present in overdense plasmas otherwise inaccessible to electromagnetic (EM) electron cyclotron (EC) waves. Among other uses, EBW can be effective for heating, driving plasma currents, and diagnostic for measurement of temperature [3]–[5]. Because EBW propagation is only possible inside the magnetized warm plasma, mode conversion from EM waves including ordinary (O) or extraordinary (X) modes [6] should be performed [7], [8]. EM particle-in-cell (EMPIC) algorithms [9]–[12] have been widely used to model collisionless kinetic systems of charged particles. One of the key steps in EMPIC simulations is the scatter algorithm, which transfers charged particle information back to the grid for subsequent use by a field solver. Scatter algorithms on irregular (unstructured) grids typically do not yield exact charge-conservation. Recently, a novel exact charge-conserving EMPIC algorithm for unstructured grids was introduced in [9], [10] based on the time-domain finite element method [13]–[15] and the exterior calculus of differential forms [16]–[21]. A variant methodology with similar properties was developed in [22], [23]. Here, we analyze dispersion characteristics of EBW propagating in the magnetized warm plasma by EMPIC algorithms on irregular grids. It is shown that the use of non-charge-conserving scatter algorithms in EMPIC simulations induces a spurious static (self-)field due to charge deposition on the grid and, as a result, produce more noisy spectral bands. The newly developed charge-conserving EMPIC solver [9], [10] is shown to produce sharper spectral bands with less noise.

II. ANALYTIC DISPERSION RELATION

In magnetized warm plasmas, there are two types of waves both propagating and polarized perpendicular to the stationary magnetic field: (i) X mode and (ii) EBW. In what follows, $k_\perp$ is the wavenumber for the perpendicular wave [rad/m], ω_{pe} is the plasma frequency [rad/s], ω_{ce} is the gyrofrequency [rad/s], ρ_{ce} is the gyroradius, and m_e is the electron mass. We assume a z-directed stationary magnetic field and EBW propagation along x. In this case, the electric field obeys the following relation (see details in [2])

$$\begin{bmatrix} 1-A & B & 0 \\ C & 1-D-n_\perp^2 & 0 \\ 0 & 0 & -n_\perp^2 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

with $n_\perp = \frac{k_\perp c}{\omega}$ and

$$A = \frac{\omega_{\mathrm{pe}}^2}{\omega} \sum_{m=-\infty}^{\infty} e^{-\lambda} \frac{m^2 I_m(\lambda)}{\lambda(\omega - m\omega_{\mathrm{ce}})}, \tag{2}$$

$$B = i\frac{\omega_{\mathrm{pe}}^2}{\omega} \sum_{m=-\infty}^{\infty} m e^{-\lambda} \frac{I_m(\lambda) - I'_m(\lambda)}{\omega - m\omega_{\mathrm{ce}}}, \tag{3}$$

$$C = -i\frac{\omega_{\mathrm{pe}}^2}{\omega} \sum_{m=-\infty}^{\infty} m e^{-\lambda} \frac{I_m(\lambda) - I'_m(\lambda)}{\omega - m\omega_{\mathrm{ce}}}, \tag{4}$$

$$D = \frac{\omega_{\mathrm{pe}}^2}{\omega} \sum_{m=-\infty}^{\infty} e^{-\lambda} \frac{\left(\frac{m^2}{\lambda} + 2\lambda\right) I_m(\lambda) - 2\lambda I'_m(\lambda)}{\omega - m\omega_{\mathrm{ce}}}. \tag{5}$$

where $\lambda = (k_\perp \rho_{\mathrm{ce}})^2/2$ and I_m is the modified Bessel function of first kind. The analytic dispersion relation is obtained by finding the zeros of the determinant in (1).

III. NUMERICAL EXAMPLE

We compare the dispersion relations for the X mode and the EBW obtained analytically as described above and by means of EMPIC simulations. The same conditions used in [24] are assumed here. Consider a magnetized warm plasma with electron density $n_{\mathrm{e}} = 2.4 \times 10^{20}$ [m^{-3}] and static applied magnetic field $\vec{B} = 5.13\hat{z}$ [T]. The electrons have initial random distribution over $[0.0005, 0.012] \times [0, 0.000025]$ and Maxwellian distribution for the thermal velocity with $|\vec{v}_{\mathrm{th}}| = 0.07c$. We set the number of simulation (macro-)particles equal to 13,800 with scaling factor 5×10^9. The motion of ions (of mass m_i)

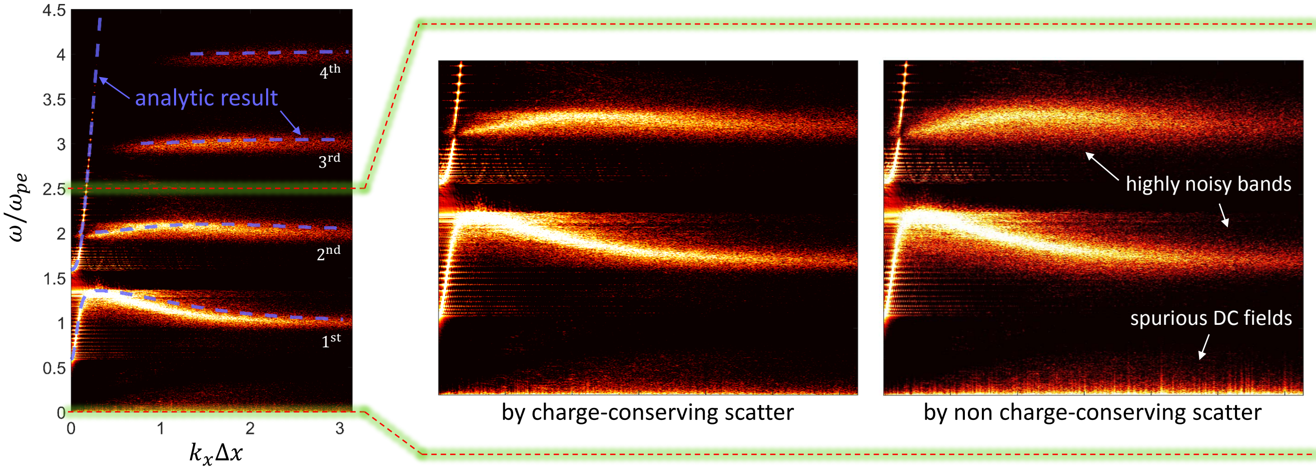

Fig. 1: Dispersion relation for X mode and EBW in the magnetized warm plasma.

is neglected since $m_i/m_e \approx 1,838$. Also, $\omega_{\rm pe} = 8.7 \times 10^{11}$ [rad/s] and $\omega_{\rm ce} = 9.0 \times 10^{11}$ [rad/s]. Using the EMPIC simulation data, we perform space and time Fourier analysis of electric field sampled in space and time to obtain the dispersion relation $\omega(k_x)$ for the X mode and the EBW. Fig. 1 shows the dispersion relations computed analytically and numerically. It can be observed that the X mode is dominant for small k_x, but as k_x increases EBW becomes dominant. The two close-in view plots compare results from the charge-conserving and non charge-conserving EMPIC simulations. In the latter case, a strong spurious static (self-)field is produced, which perturbs the particle trajectories and is evidenced by the noisy spectral bands. Note that the absolute spectral resolution is affected by the time interval employed in the EMPIC simulation. We have chosen identical time intervals for both simulations. It is expected that the difference between the charge-conserving and non charge-conserving cases should become even more significant if longer time intervals are used.

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