# Warm Plasma Raytracing of Whistler Mode Waves in the Earth's Magnetosphere

Ashanthi Maxworth, Mark Golkowski
Department of Electrical Engineering
University of Colorado Denver
Denver, Colorado, USA
Ashanthi.Maxworth@ucdenver.edu, Mark.Golkowski@ucdenver.edu

*Abstract*—**Whistler mode waves play a major role in the energy dynamics of the Earth's upper atmosphere. Previous studies show that inclusion of finite electron and ion temperature can modify the refractive index surface significantly for frequencies near the lower hybrid resonance and hence modify the ray trajectories. In this work we further study the properties of whistler mode waves originating around L =4, with and without finite temperature effects. According to our ray tracing results inclusion of finite temperature increases the number of magnetospheric reflections. It also confines the wave energy inside the plasmasphere. Agreement of the ray tracing results and the Van Allen Probes observations are also presented.**

## I. Introduction

Whistler mode wave propagation in the Earth's magnetosphere plays a major role in its energy dynamics. The majority of previous works in the published literature were conducted under the cold plasma assumption for the background plasma in the magnetosphere. However, the Earth's magnetosphere has known temperatures in the range of few electron volts (eVs). In 2015 [3] studied the modifications to the refractive index surface when finite temperature effects are taken into account. In their work they observed significant modifications to the refractive index surface closer to the lower hybrid resonance frequency.

In our preliminary results presented at [4],we were able to analyze the effect of temperature on the whistler mode wave trajectories, with numerical raytracing. Numerical raytracing is the process of determining the power flow path of a wave by solving the Haselgrove equations [2] at each time step. Our initial raytracing observations were as follows;

- for wave frequencies few kHz less than the lower hybrid resonance frequency, inclusion of temperature does not modify the wave trajectory.
- if the wave frequency is close to the lower hybrid resonance frequency, inclusion of ion temperature plays a major role. Inclusion of ion temperature closes an otherwise open refractive index surface as shown in [3]. Hence inclusion of ion temperature increases the number of magnetospheric reflections (MRs).
- when the wave frequency is few kHz higher than the lower hybrid resonance frequency, inclusion of electron temperature introduces more reflections. But this modifications is not significant compared to the observation made for frequencies close to the lower hybrid resonance frequency.

Apart from the above observations we also commented on behavior of the refractive index surface when temperature effects were given to both electrons and ions. Our main observation was inclusion of a 1eV temperature to both electrons and ions produces the smallest refractive index surface. Hence adding a temperature to both species, closes the refractive index surface most tightly. Therefore in this work we include a temperature to both electrons and ions. Also in order to study the MLT dependence of the waves and also to increase the accuracy of the simulations we have used the MLT dependent temperature values given in [1].

## II. Technical Approach

In this work we were using the Stanford 3D Raytracer for the simulation purposes. As mentioned above the raytrajectory was determined by solving the dispersion relation equation at each step. For cold plasma the order to the dispersion relation is four. With the inclusion of temperature the order of the dispersion relation increases to six. The dispersion relation for warm plasma is modified as follows when both electron and ion temperature are taken into account.

$$\Sigma_s q_s^T A_{1s}\mu^6+(A_0+\Sigma_s q_s^T B_{1s})\mu^4+(B_0+\Sigma_s q_s^T C_{1s})\mu^2+C_0=0. \tag{1}$$

$$q_s^T=\frac{k_B T_s}{m_s c^2} \tag{2}$$

In Equation (1),$\mu$ is the refractive index, the parameters $A_1, B_1$ and $C_1$ are warm plasma parameters that are functions of wave normal angle, plasma frequency and cyclotron frequency. The parameters $A_0, B_0$ and $C_0$ are cold plasma parameters that are functions of wave normal angle and Stix parameters.The parameter $q^T$ is a function of temperature, mass of the particle and the density as given in Equation (2). In Equation (2) $k_B$ is the Boltzmann constant, $T$ and $m$ are the temperature and mass respectively and the subscript $s$ indicate different species, $c$ is the speed of light. Further details of the formulation are provided in [3].

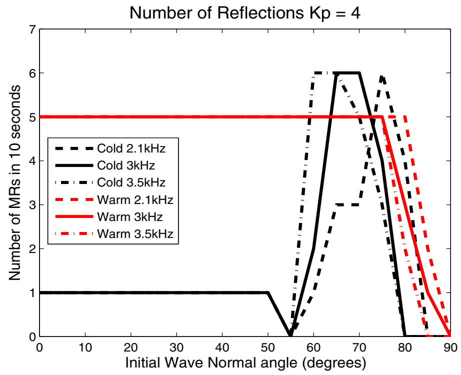


Fig. 1. Number of reflections observed within a duration of 10 seconds for whistler mode waves launched at $L = 4$ for $k_P = 4$

## III. Results and Conclusions

Figure 1 shows the number of observed reflections for whistler mode waves launched at $L = 4$, with respect to frequency. The waves were launched with three selected initial wave normal angles in order to capture the full behavior. All waves were simulated for a duration of 10 seconds. As depicted in Figure 1 under cold plasma assumptions, the waves showed either no or very few number of reflections. In most of the cases the waves were either guided by a geomagnetic field line or exhibited only one reflection before hitting the surface of the Earth. Whereas under warm plasma assumptions, the waves underwent multiple magnetospheric reflections within a duration of 10s.

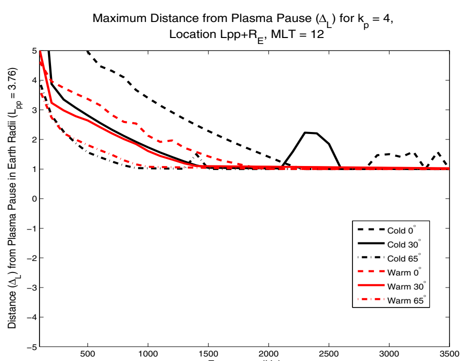


Fig. 2. Maximum distance($\Delta_L$) in Earth radii, from the location of the plasmapause. In this case the plasmapause was at $L = 3.76$. The duration of wave propagation was 10 seconds.The waves were launched at $L_{pp} + 1$

In order to study the effect of the source location of the waves, we have selected two source locations. One source location was one Earth radii inside the plasma pause and one source location was one Earth radii outside the plasma pause. And we observed the maximum displacement of the waves from the plasma pause within a propagation duration of 10 seconds. Figure 2 shows the maximum displacement of the waves from the plasma pause for the originating location $L_{pp}+1$, with respect to wave frequency. From the observations in Figure 2, warm plasma results show a much more consistent behavior compared to the cold plasma results. According to the simulation results given in Figure 2, under warm plasma assumptions wave energy confines to a region around one Earth radii from the plasma pause, which shows a reasonable agreement with the Van Allen probe observations. A similar plot was obtained when the waves were launched one Earth radii inside the plasma pause.

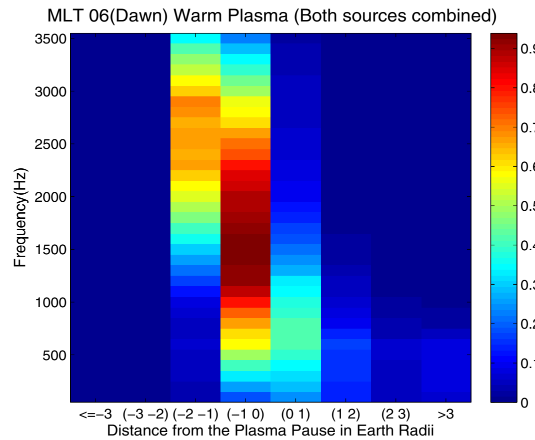


Fig. 3. Maximum distance($\Delta_L$) in Earth radii, from the location of the plasmapause. In this case the plasmapause was at $L = 3.76$. The duration of wave propagation was 10 seconds.The waves were launched at two source locations $L_{pp} - 1$ and $L_{pp} + 1$

Figure 3 shows the normalized combined power flow through each region in space, due to two magnetospheric sources each at one Earth radii inside and outside the plasma pause. According to Figure 3 for frequencies in the range of $500Hz - 2kHz$ the wave power is confined inside the plasmasphere around $1R_E$ from the plasma pause. Also as the frequency increases the maximum power confinement is being shifted towards the Earth. This result is in direct agreement with the Van Allen probe data observed at dawn (MLT =06). Such a close agreement was not observable under cold plasma assumptions. Landau damping is assessed as the next step of this work.

## Acknowledgement

Authors would like to acknowledge the support from Dr. David Malaspina and Dr. Allison Jaynes at the Laboratory of Atmospheric and Space Physics, Boulder CO.

## References

[1] P. M. E. Decreau, C. Beghin and M. Parret, Global characteristics of the cold plasma in the equatorial plasmasphere region as deduced from the GEOS 1 mutial impedance probe. *J. Geophys. Res A87,695* 1982

[2] J. Haselgrove, Ray Theory and a New Method for Ray Tracing, in Physics of the Ionosphere,*Physics of the Ionosphere Conference, Cambridge, London, 355-364*, 1954

[3] P. Kulkarni, M.Golkowski,U.S.Inan and T.F.Bell, The effect of electron and ion temperature on the refractive index surface of 110 kHz whistler mode waves in the innermagnetosphere, *Geophys. Res. Space Physics, 120, 581591*,2015

[4] A.S.Maxworth, M. Golkowski, Effect of finite electron and ion temperature on magnetospheric whistler mode raytracing, *USNC-URSI NRSM 2016, Boulder, CO*, doi: 10.1109/USNC-URSI-NRSM.2016.7436225