A Rayleigh-Ritz Approach to Green's Function of an Inhomogeneous Layer

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Nature is not homogeneous, and layered structures are encountered in numerous disciplines such as in the study of oceans, atmosphere, earth sciences, and ionosphere. Besides there is a host of man-made materials and devices where carefully designed inhomogeneities are an essential part of their function. In all such problems the Green's function concept is very useful for theoretical analysis and development.

For vertically stratified media one can use transform methods to formally represent the Green's function in terms of depth dependent solutions. However, the depth dependent solutions are available only for piece-wise homogeneous problems. For more general problems the solutions may be obtained formally by wave-number integration techniques. Because of slow convergence and singularities this computation is not simple. More general numerical methods that produce accurate results are finite difference method, finite element method, and the method of moments. On the other hand, there are approximate methods that can lead to fairly good results with far less computational effort. The WKB method is useful for high frequencies, and for smooth and large scale structure of inhomogeneities. Parabolic equation approximation is a well-established method in problems where the scattered energy is essentially in the forward direction.

A more simple and general method that involves considerably less effort is the Rayleigh-Ritz (RR) approach. Here a variational formulation equivalent to the boundary value problem of the inhomogeneous medium is employed. Since the differential operator for this problem is self-adjoint, there exists a functional, the variation of which leads to the original boundary value problem. The wave function is expressed in terms of the mode functions of the corresponding homogeneous problem. The stationarity condition leads to a matrix equation, whose solution gives the coefficients of the mode functions.

One important question is about the accuracy of the RR method. Convergence and stability are other issues. These are rather technical questions which are difficult to be answered in a general situation. Instead we have used examples with known solutions for assessing the RR method. For a few special inhomogeneous structures we have constructed the exact Green's functions, and carried out detailed study of the RR method for these cases. Based on this study we find that the RR method is quite accurate for low and medium frequencies, when the amplitude of inhomogeneity is not large. For more general situations, we have developed a finite element approach to generate benchmark solutions. We have used this to study the RR approach to problems where the inhomogeneous profile has inflection points. From the study thus far we have found that the RR method is simple, convenient, and fairly general for constructing Green's functions of inhomogeneous layered problems.