Non-Causal Filtering Applied to Numerical Whistler Mode Raytracing

Ashanthi Maxworth, Titsa Papantoni, Mark Golkowski,
Department of Electrical Engineering
University of Colorado Denver
Denver, CO, USA
ashanthi.maxworth@ucdenver.edu , titsa.papantoni@ucdenver.edu, mark.golkowski@ucdenver.edu,

***Abstract*—Non causal filtering or smoothing is an important signal processing technique. In this work we implement a new non causal filter and apply it to whistler mode ray tracing platform. This smoothing technique is highly resistant to outliers; hence it helps on increasing the accuracy of the numerical results.**

I. INTRODUCTION

Non-causal filtering or smoothing is an important signal processing technique. Due to its advantages over causal filtering, it has been applied in the fields on acoustic engineering and image processing [2]. In this work we apply a non-causal filtering technique developed for stochastic signals presented by [3], to whistler mode ray tracing. This is the first time implementation of the Kazakos, and Papantoni-Kazakos [3] non-causal filtering technique

Whistler mode waves are a type of very low frequency electromagnetic waves in the Earth's magnetosphere. The Earth's magnetosphere is in a plasma state and whistler mode waves are an electromagnetic plasma wave with very intense amplitudes sometimes on the order of 100 dB-pT Being very intense and prevalent in nature whistler mode waves drive nonlinear interactions in the magnetospheric plasma. These interactions form an important component of space weather and are associated with damage to space electronics and disturbunaces in space communication systems. Therefore it is important to understand the behavior of whistler waves and determine the propagation path of those waves.

Computing the trajectories of whistler mode waves is known as ray tracing and bears many similarities to geometric optics. Raytracing identifies the power flow path of a wave by solving the Haselgrove equations [1]. Ray tracing involves discretizing the magnetosphere and calculating the local refractive index at the wave location. The refractive index provides the group velocity magnitude and direction so that the subsequent position of the wave energy can be determined and a ray-path can be traced.

In this work we used the Stanford 3D raytracer for raytracing.

The original Stanford 3D ray tracer was modified accordingly in order to take the temperature effects into account, considering real magnetospheric environment. The full formulations for raytracing can be found in [4].

$$\mu = \frac{kc}{\omega} \quad (1)$$

Refractive index and the wave normal vector **k** are related according to equation (1). In equation (1) ω, is the angular frequency of the wave and c is the speed of light. In the Stanford 3D raytracer, the maximum difference (also named as maximum error) between the magnitude of the **k** vector consecutive locations can be set by the user. If this value is too high the difference between two consecutive refractive indices will be large hence the predicted curve will be a rough approximation due to its high probability of outlier occurrence. On the other hand if this value is too small the predicted ray trajectory will be very smooth, but the simulation time will increase drastically. Hence it is important to find a nominal value for the maximum difference between two consecutive k vector magnitudes, without encountering outliers.

The method we followed was as follows; we run the simulations with a very small error (~0.001) for multiple times. Then we run the simulations with a nominal value (~0.01) and find the upper and lower limits of the outlier amplitudes. The outlier amplitude is named as the truncation constantλ_n, in the following section. In subsequent runs with the nominal error if the generated k value magnitude goes beyond the truncation constant then the simulation, refines it's time step such that the truncation constant is not exceeded.

II. TECHNICAL APPROACH

This section explains the functionality of the smoother as it appears in [3].

For Gaussian density functions f_{0s} and f_{0N} we select some probability of outlier occurrence $\varepsilon_N(0,1)$ and some finite nonnegative integer m . Let $\{\dots\dots X_{-1,} X_0, X_1 \dots\dots\}$ and $\{\dots\dots W_{-1,} W_0, W_1 \dots\dots\}$ denote sequences of random variables that are generated by the above Gaussian density functions f_{0s} and f_{0N} respectively. Given some integer k and some nonnegative integer n , let $N_{2n+1,k}$ and $M_{2n+1,k}$ respectively denote the auto- covariance matrices, $E\{W_{k-n}^{k+n}(W_{k-n}^{k+n})^T|f_{0N}\}$ and $E\{X_{k-n}^{k+n}(X_{k-n}^{k+n})^T|f_{0s}\}$. Let $a^T{}_{2n+1,k}$ denote the $(n+1)^{\text{th}}$ row of the matrix $M_{2n+1,k}$.

$$\Lambda_{2n+1,k} = M_{2n+1,k} + N_{2n+1,k} \quad (2)$$

And let $g_{kl}^{0}\left(x_{k-n}^{k-l}, x_{k+l}^{k+n}\right); n \geq l$ denote the optimal mean-squared interpolation operation at the Gaussian density function f_{0s} for the datum x_k, given x_{k-n}^{k-l}, and x_{k+l}^{k+n}. Let us then define the sets $\{d_{k,n,l,j}; k-n \leq j \leq k-l, k+l \leq j \leq k+n\}$ and $\{b_{k,n,j}; k-n \leq j \leq k+n\}$ of coefficients as follows, where $\Lambda_{2n+1,k}$ is assumed nonsingular.

$$\{d_{k,n,l,j}\}: g_{kl}^{0}\left(x_{k-n}^{k-l}, x_{k+l}^{k+n}\right) = \sum_{j=k-n}^{k-l} d_{k,n,l,j} x_j + \sum_{j=k+l}^{k+n} d_{k,n,l,j} x_j \qquad (3)$$

$$\left[b_{k,n,k-n} \dots\dots\dots\dots\dots\dots b_{k,n,k+n}\right] = a^{T}{}_{2n+1,k}\Lambda_{2n+1,k}{}^{-1} \qquad (4)$$

Let us now define

$$g_n^s(x) = \begin{cases} x & \text{if } |x| \leq \lambda_n \\ \lambda_n sgn(x) & \text{otherwise} \end{cases} \qquad (5)$$

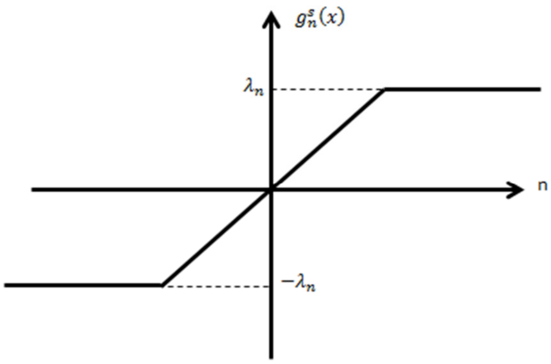

Fig. 1: shows the smoothing process as given by equation (5).

Where $c = \lambda_n [a^{T}{}_{2n+1,k}\Lambda_{2n+1,k}{}^{-1} a_{2n+1,k}]^{-1/2}$ such that

$$\Phi(c) + c^{-1}\phi(c) = 2^{-1}[1 + (1 - \varepsilon_N)^{-1}] \qquad (6)$$

Let $\hat{x}_{k,n}^{s}$ denote the estimate of the signal datum x_k from the observation vector y_{k-n}^{k+n}. Then the estimate $\hat{x}_{k,n}^{s}$ is designed as;

$$\hat{x}_{k,n}^{s} = \begin{cases} g_n^s\left(a^{T}{}_{2n+1,k}\Lambda_{2n+1,k}{}^{-1} y_{k-n}^{k+n}\right) & \text{if } n \leq m \\ g_{kl}^{0}\left(x_{k-n}^{k-l}, x_{k+l}^{k+n}\right) & n > m \end{cases} \qquad (7)$$

Where $\hat{x}_j^{i} = [\hat{x}_{j,m}^{i} \dots\dots\dots \hat{x}_{j,m}^{s}]$; i > j

Let us define

$$r_n^s(n) = a^{T}{}_{2n+1,k}\Lambda_{2n+1,k}{}^{-1} a_{2n+1,k} \qquad (8)$$

Then $r_k^s(n)$ represents a variance gain in estimating the signal datum x_k from the observation vector y_{k-n}^{k+n} at the zero mean Gaussian noise density

Therefore $r_k^s(n)$ is monotonically non-decreasing with n. Given ε_N, the same monotonicity characterizes the truncation constant λ_n, whose maximum value λ_∞ equals to c $\lim_{n\to\infty}[r_k^s(n)]^{1/2}$, where c is the solution to the equation (5).

III. Results and Conclusions

By analyzing the signal's probability distribution function of the output data sequence, we have selected the probability of outlier occurrence ε_N, to be 0.05. The length of the observation data sequence n, was selected as 1000, and the output data sequences were observed 2001 times, hence k was 2001. The results are shown in Figure 2.

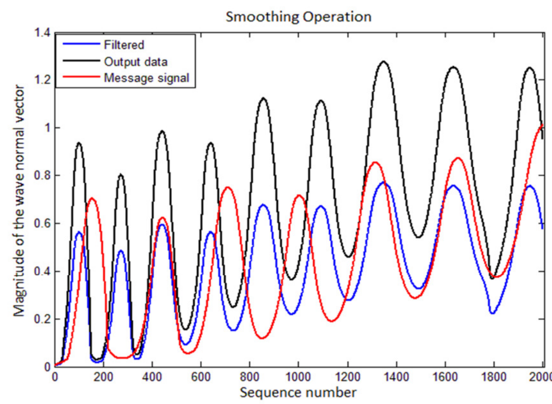

Fig. 2: plot of data sequences. Red: the message sequence or the data sequence generated with a minimal error. Black: The output data sequence with the nominal error. Blue: Output data sequence after filtering.

The noise process was assumed to be zero mean and Gaussian. Results shown in Figure 2 show the effectiveness of smoothing. It is clear that after the smoothing process, that the filtered data sequence lies closer to the mean of the signal, whereas the original output signal has a higher standard deviation. The average value for the truncation constant λ_n was found to be 2.15.

Acknowledgement

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