Polarimetric Interference Alignment in MIMO Broadcast Channels

Carlos A. Viteri-Mera*† and Fernando L. Teixeira*
*ElectroScience Laboratory, The Ohio State University, Columbus, OH, USA
†Universidad de Nariño, Pasto, Colombia
{viteri.5, teixeira.5}@osu.edu

***Abstract*—We investigate the concept of polarimetric interference-alignment (PIA) applied for MIMO broadcast channels. PIA takes advantage of the orthogonal polarizations of electromagnetic waves to provide a signal space for interference-alignment. We provide a preliminary discussion on its operation and performance applied to multiuser-MIMO systems.**

I. Introduction

Interference alignment (IA) is a signal processing concept with origins in the information theoretic analysis of interference channels. It achieves the maximum number of degrees of freedom in K-user interference channels [1]. Consider a wireles system with K transmitter-receiver pairs, where every terminal has an arbitrary number of antennas. The basic idea of IA is to separate the desired signal and interference *at each receiver* into orthogonal subspaces [2]. By using IA, every trasmitter-receiver pair in the interference channel can achieve half the capacity they would achieve under no interference (independent of the number of users) [1], [3]. In most cases, IA schemes use a combination of space (antennas) and time dimensions to perform the alignment, which demand increasing bandwidth as the number of users increases. A number of works have focused on spatial IA, which uses the spatial dimension available in MIMO communication systems to perform IA. This is done by using linear precoding and postprocessing (transmitter and receiver beamformers), as seen for example in [4]–[6]. In these spatial alignment schemes, coding across time or frequency slots is not required. Recently, we propose implementing IA over the field *polarization* components using a linear precoder [7], [8], an approach we denote *polarimetric interference alignment* (PIA). In this paper, we look at the IA concept applied to MIMO broadcast channels where a single access point (AP) or base station sends information to multiple users. We provide a summary of the formulation and some preliminary results.

II. System Model

Consider a baseband wireless communication system consisting of one AP with M transmit vector antennas and K user terminals, as depicted in Fig. 1. Each user terminal is equipped with a single vector antenna. We assume that each vector antenna at the transmitter (receiver) can generate (detect) N_t (N_r) orthogonal polarization components. The system operates in TDD mode, i.e., the uplink and the downlink use the same frequency bands in different time slots. Thus, using reciprocity [9], any channel estimate can be used in both uplink and downlink. The PIA procedure is divided into two stages: *i*) channel estimation, and *ii*) transmission, as detailed next.

A. Channel Estimation Phase

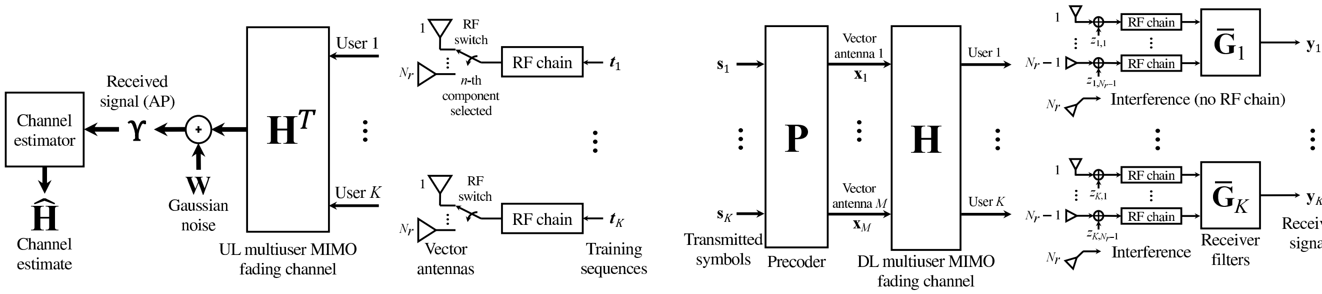
Fig. 1a depicts the channel estimation phase. We assume that the system operates in a flat fading channel, where propagation between each transmitter/receiver antenna pair is modeled with a complex channel coefficient. The downlink channel matrix $\mathbf{H} \in \mathbb{C}^{KN_r \times MN_t}$, which relates each polarization component of the transmit antennas (MN_t inputs) with each polarization component in the receiving antennas (KN_r outputs), characterizes the uplink channel. Note that we can define the stacking $\mathbf{H} = \left[\mathbf{H}_1^T \cdots \mathbf{H}_K^T\right]^T$, where $\mathbf{H}_k \in \mathbb{C}^{N_r \times MN_t}$ is the channel matrix for user k. The AP can obtain an estimate $\hat{\mathbf{H}}$ of the channel matrix using a training sequence transmitted from every user and applying a linear estimator to the received signal [10]. One RF chain per user is required for the estimation in the uplink, since an RF switch can sweep the training signal across the user antennas.

B. Transmission

At a given sampling instant, the transmitter sends independent signals simultaneously to all users. Let $\mathbf{s}_k \in \mathbb{C}^{N_r-1}$ be the vector of complex transmitted symbols with N_r-1 spatial streams (degrees of freedom) directed to user k. The remaining user antenna, which we arbitrarily select as the N_r-th antenna (polarization component), is used to align all interference and does not have an RF chain in the downlink. Thus, PIA uses the uplink channel estimate from N_r antennas but only transmits N_r-1 streams in the downlink, aligning inter-user interference in the unobservable N_r-th signal dimension at the receiver. In this way, the downlink is represented with K point-to-point interference-free MIMO links with a total of MN_t antennas at the transmitter and N_r-1 antennas at the receivers. The received signal at user k is

$$\mathbf{y}_k = \underbrace{\mathbf{G}_k\mathbf{H}_k\mathbf{P}_k\mathbf{s}_k}_{\text{Desired signal}} + \underbrace{\sum_{\substack{k'=1\\k'\neq k}}^{K} \mathbf{G}_k\mathbf{H}_k\mathbf{P}_{k'}\mathbf{s}_{k'}}_{\text{Interference}} + \underbrace{\mathbf{G}_k\mathbf{z}_k}_{\text{Noise}} \in \mathbb{C}^{N_r-1},$$

where $\mathbf{P}_k \in \mathbb{C}^{(MN_t)\times(N_r-1)}$ is the precoder for user k, and $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \eta\mathbf{I}_{N_r})$ is the Gaussian noise vector at



(a) (b)

Fig. 1. (a) Uplink and (b) downlink system models for polarimetric interference alignment.

the receiving antennas. $\mathbf{G}_k$ represents the linear processing applied at the receiver defined as $\mathbf{G}_k = \left[\bar{\mathbf{G}}_k \mathbf{0}_{(N_r-1)\times 1}\right] \in \mathbb{C}^{(N_r-1)\times N_r}$, where $\bar{\mathbf{G}}_k \in \mathbb{C}^{(N_r-1)\times(N_r-1)}$ is the receiver beamformer, and the zero padding in the last column represents the absence of an RF chain for the N_r-th antenna (polarization component) in the downlink. We also define the precoding matrix $\mathbf{P} = [\mathbf{P}_1 \cdots \mathbf{P}_K] \in \mathbb{C}^{MN_t\times K(N_r-1)}$. The idea behind PIA is to focus the desired signal in the first N_r-1 received signal dimensions (all observable at the receiver), while aligning the interference in the last dimension. Thus, if we focus on the precoder design and set $\bar{\mathbf{G}}_k = \mathbf{I}$ (no receiver processing), we can establish the following conditions for IA: $\hat{\mathbf{H}}_k \mathbf{P}_k = \left[\bar{\mathbf{\Sigma}}_k \ \mathbf{0}_{(N_r-1)\times 1}\right]^T = \mathbf{\Sigma}_k$, $\forall k$ and, $\hat{\mathbf{H}}_k \mathbf{P}_{k'} = \left[\mathbf{0}_{(N_r-1)\times(N_r-1)} \ \boldsymbol{\beta}_{k,k'}\right]^T = \mathbf{B}_{k,k'}$, $\forall k \neq k'$, where $\boldsymbol{\beta}_{k,k'} \in \mathbb{C}^{N_r-1}$ is an arbitrary weight vector for the discarded interference, and $\bar{\mathbf{\Sigma}}_k$ is a diagonal matrix with the spatial stream weights. These PIA conditions can be written in matrix form as

$$\mathbf{P} = \hat{\mathbf{H}}^H \left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H\right)^{-1} \begin{bmatrix} \mathbf{\Sigma}_1 & \mathbf{B}_{2,1} & \cdots & \mathbf{B}_{K,1} \\ \mathbf{B}_{1,2} & \mathbf{\Sigma}_2 & \cdots & \mathbf{B}_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{1,K} & \mathbf{B}_{2,K} & \cdots & \mathbf{\Sigma}_K \end{bmatrix},$$

where the matrix in the right is calculated such that $\|\mathbf{P}\|_F^2 = 1$.

III. Preliminary Results and Discussion

We simulated a PIA system using the parameters in Fig. 2. We assumed perfect estimation at the transmitter $\hat{\mathbf{H}} = \mathbf{H}$, and that the entries of $\mathbf{H}$ are complex circularly-symmetric uncorrelated random variables with zero mean and unit variance. We compared the PIA precoder performance in terms of the ergodic achievable sum-rate over 10^4 channel realizations versus block-diagonalization (BD) [11] and maximum ratio transmission (MRT). Fig. 2 shows that PIA performs below BD but improves MRT in the high SNR regime. PIA performance is also closer to BD when the number of antennas at the receiver increases. There are still open questions regarding the optimization of PIA model parameters that could improve its performance.

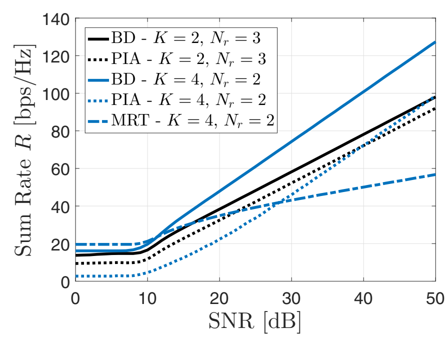


Fig. 2. Achievable sum-rate of PIA, BD and MRT with $M = 4$, and $N_t = 3$.

References

[1] V. Cadambe and S. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug 2008.

[2] S. A. Jafar, *Interference Alignment: A New Look at Signal Dimensions in a Communication Network*. Now Publishers Inc., 2011.

[3] M. Morales-Cespedes *et al.*, "Blind Interference Alignment for Cellular Networks," *IEEE Trans. Signal Process.*, vol. 63, no. 1, pp. 41–56, Jan 2015.

[4] S. Peters and R. Heath, "Interference Alignment via Alternating Minimization," in *IEEE Int. Conf. Acous., Speech and Signal Proc. (ICASSP)*, April 2009, pp. 2445–2448.

[5] K. Gomadam, V. Cadambe, and S. Jafar, "A Distributed Numerical Approach to Interference Alignment and Applications to Wireless Interference Networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, June 2011.

[6] M. Razaviyayn, M. Sanjabi, and Z.-Q. Luo, "Linear Transceiver Design for Interference Alignment: Complexity and Computation," *IEEE Trans. Inf. Theory*, vol. 58, no. 5, pp. 2896–2910, May 2012.

[7] C. A. Viteri-Mera and F. L. Teixeira, "Polarimetric Interference-Alignment Beamforming," in *IEEE Int. Symp. Antennas Propagat.*, June 2016, pp. 533–534.

[8] ——, "Feasibility Analysis of Polarimetric-Interference Alignment Beamforming in Rich-Scattering Indoor Channels," in *IEEE Int. Symp. Antennas Propagat.*, June 2016, pp. 335–336.

[9] M. E. Yavuz and F. L. Teixeira, "Ultrawideband Microwave Sensing and Imaging Using Time-Reversal Techniques: A Review," *Remote Sens.*, vol. 1, no. 3, p. 466, 2009.

[10] M. Biguesh and A. B. Gershman, "Training-Based MIMO Channel Estimation: A Study of Estimator Tradeoffs and Optimal Training Signals," *IEEE Trans. Signal Process*, vol. 54, no. 3, pp. 884–893, March 2006.

[11] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels," *IEEE Trans. Signal Proc.*, vol. 52, no. 2, pp. 461–471, February 2004.