Surface Integral Equation Formulation of Electromagnetic Scattering for Cloaking Applications

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We investigate the role of the electric field and its normal derivative in three-dimensional electromagnetic scattering theory. In particular, we present an alternative integral equation formulation that uses the electric field and its normal derivative as the boundary unknowns. In particular, we extend a traditional formulation that is used in two-dimensional scattering theory to three-dimensions. Our formalism may be advantageous in some avant-garde applications such as cloaking (invisibility) devices that have fascinated minds for decades, if not centuries. It is well-known that the boundary conditions have an extensive effect on the solution, and thus it is not surprising that unorthodox boundary conditions provide the most elegant means for achieving a solution. Indeed, for cloaking applications, such unorthodox boundary conditions on the normal components of the electromagnetic field or its derivative, rather than on the tangential components, provide the required solution. In particular, it has been shown that the vanishing of these normal components is required to achieve some cloaking aspects (R. Weder, J. Phys. A: Math. Theor., 41, 415401, 2008 and I.V. Lindell and A.H. Sihvola, IEEE Trans. Antennas Propag., 58, 1128–1135, 2010).

The presentation begins with a derivation of the continuity conditions for the electric field and its normal derivative across an interface. From these conditions, we uncover several intriguing relationships involving closed surface integrals of the field and/or its normal derivative. For example, we show that

$$\int_{\Sigma} \boldsymbol{N} \cdot \frac{\partial \boldsymbol{E}}{\partial N} \, \mathrm{d}S = 0$$

for closed surfaces with a zero mean curvature. We end the presentation with a physical interpretation of the surface integrals appearing in the above discussed integral equation formulation. In order not to obscure the physical/geometric awareness, the presentation is given from a tensor-calculus perspective.