Quality Factor Calculations for the Characteristic Modes of Dielectric Resonator Antennas

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***Abstract*— Characteristic mode theory (CMT) is employed to calculate the eigenmodes of dielectric objects based on the volume integral equation. With the knowledge of modal current, the quality factor (Q) of the characteristic modes are then calculated. In contrast to most conventional analysis techniques, the modal Qs are now available at any frequency, rather than only their resonant frequencies. This method offers additional information that can be used for single and multi-port dielectric resonator antenna (DRA) design and shape optimization. To verify our method, a cylindrical dielectric resonator is studied, and the Q factors at its natural resonance frequencies are compared with results from previous literature. This approach can be readily applied to geometries of arbitrary shape.**

I. Introduction

Dielectric resonator antennas (DRA) are an important type of highly efficient antennas at microwave and millimeter wave frequencies. Like most resonant antennas, one of the important properties of a DRA is its quality factor (Q). The Q can be calculated analytically for canonical shapes (*e.g.*, sphere, cylinder, rectangular prism), but for DRAs of arbitrary shape, calculation of Q relies on numerical methods or measurement. Using conventional numerical methods [1], the Q factors can be captured only at the natural resonance frequencies by finding the complex frequencies at which the operator matrix has a zero determinant. This method provides no information about the modal Q of the DRA at frequencies other than these natural resonances. Still, modes can be excited and matched off-resonance with careful feed design, so their properties may hold important insight into performance of more advanced DRAs.

Here we revisit the calculation of DRA Q factors using characteristic modes theory (CMT) [2], which solves for the eigenmodes of an arbitrary radiating structure. We show that CMT is a suitable tool, if not the only one, to capture the modal behaviors of arbitrary structures over a wide frequency range. After the characteristic mode analysis, the Q of each mode is calculated from the modal current based on the formulation in [3]. Using this approach, we study a cylindrical DRA, for which results are readily available in literature for comparison.

II. Numerical Formulation

The application of CMT requires generation of an impedance matrix via the method of moments. When solving a problem involving complex dielectric objects using the MoM, formulations are typically based the surface integral equation (SIE) or the volume integral equation (VIE). Though the SIE has advantages over the VIE in terms of computational complexity, it suffers from the problem of non-physical modes [4]. It is also not clear whether the source formulation discussed in Section II-A can be applied to the SIE due to the non-physical internal fields in the equivalent problem. Thus, we employ the VIE to study the characteristic modes and the modal Qs of DRA. When using mixed potentials to express the scattered field, the VIE formulation has the following well known form [5]

$$\frac{\boldsymbol{J}(r)}{j\omega(\hat{\epsilon}(\boldsymbol{r})-\epsilon_0)} + j\omega\boldsymbol{A}(\boldsymbol{r}) + \nabla\Phi(\boldsymbol{r}) = \boldsymbol{E}^i(\boldsymbol{r}) \qquad (1)$$

where $\hat{\epsilon}(\boldsymbol{r}) = \epsilon(\boldsymbol{r}) - j\sigma(\boldsymbol{r})/\omega$, reflects the material properties of the DRA and $\boldsymbol{A}(\boldsymbol{r})$ and $\Phi(\boldsymbol{r})$ are respectively the vector magnetic potential and scalar electric potential.

Both full and half SWG basis functions [5] are used for the basis expansion, with the full SWG basis modeling the volume current of the homogeneous region and the half SWG basis modeling the dielectric and air interface. Here for the convenience of Q calculation, the electric polarization current $\boldsymbol{J}(\boldsymbol{r})$ is modeled as unknowns instead of the electric flux density. The current analysis is limited to homogeneous dielectric objects, though they may be of arbitrary shape.

A. Characteristic Modal Q Calculation

Once the MoM Z matrix is obtained from above formulation, CM analysis is conducted by solving the generalized eigenvalue equation

$$\boldsymbol{X}J_n = \lambda_n \boldsymbol{R}J_n \qquad (2)$$

where $\boldsymbol{X}$ and $\boldsymbol{R}$ are the imaginary and real parts of the MoM Z matrix, and J_n and λ_n are the eigenvector and eigenvalue of the *n*th mode.

With the knowledge of modal currents, the Q factor of each mode can be calculated using the source formulation derived by Vandenbosch [3]. Considering only loss from radiation, the Q factor of each mode of the DRA is defined as

$$Q_{rad} = \frac{2\omega_0 \max\{W^e, W^m\}}{P_{rad}} = \frac{2\omega_0 \max\{W^e_{vac}+W^e_{mat}, W^m_{vac}\}}{P_{rad}} \qquad (3)$$

where W^e_{vac} and W^m_{vac} are the stored electric and magnetic energy in free space, W^e_{mat} the stored electric energy in material, and P_{rad} the radiated power. W^e_{vac}, W^e_{mat}, W^m_{vac} and P_{rad} can be evaluated using the current and charge density within the dielectric object. In the approach found in [3], there are two types of integrals to be evaluated for the stored energy calculations:

$$I_1 = \int_{V_1}\int_{V_2} q_1(\boldsymbol{r_1}) q_2^*(\boldsymbol{r_2})\, g(\boldsymbol{r_1},\boldsymbol{r_2}) dV_1 dV_2 \qquad (4)$$

and

$$I_2 = \int_{V_1}\int_{V_2} \boldsymbol{J}_1(\boldsymbol{r_1}) \cdot \boldsymbol{J}_2^*(\boldsymbol{r_2})\, g(\boldsymbol{r_1},\boldsymbol{r_2}) dV_1 dV_2 \qquad (5)$$

where $g(\boldsymbol{r_1},\boldsymbol{r_2})$ could be $\frac{\cos(k_0 r_{21})}{r_{21}}$, $\frac{\sin(k_0 r_{21})}{r_{21}}$ or $\sin(k_0 r_{21})$, depending on which energy or power is calculated, and charge density q is defined as $q = \nabla \cdot \boldsymbol{J}$.

However, there are two types of charges (volume and surface) inside the dielectric objects. Therefore, there are four types of terms instead of one when evaluating the reaction between the charges inside the object: volume-volume, volume-surface, surface- volume and surface-surface.

III. Results

To verify our methodology, an isolated cylindrical dielectric resonator with measured Q in literature [6] is revisited here. The cylinder ($\epsilon_r = 38$) has a diameter of 12.83 mm and height of 5.62 mm. The meshed geometry is shown in Figure 1(a).

CM analysis of the cylindrical dielectric resonator is conducted over the frequency range of 3.5 GHz to 6.5 GHz. The modal significance ($MS = \frac{1}{|1+j\lambda_n|}$) of each mode is shown in Figure 1(b), with their natural resonances occurring at the modal significance peaks. The Q factors of the first few modes are calculated and shown in Figure 1(c) with their natural resonance points marked with a dot. Table I compares both the calculated resonant frequencies and the Q factors at the natural resonance frequencies with those from literature [6]. Both the resonant frequencies and the modal Qs are quite close to the measurement results, demonstrating the robustness of the proposed method. However, it is clear that our approach provides the modal Q factors at all frequencies, offering more insights on antenna design. For example, Fig. 1(c) shows that although mode $TE_{01\delta}$ resonates at the lowest frequency, it does not have the lowest Q factor. In fact, both $TM_{01\delta}$ and $HEM_{12\delta}$ have lower Q factors over much of the band, indicating that these could provide more bandwidth than $TE_{01\delta}$ mode if they can be excited.

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TABLE I
CYLINDRICAL DIELECTRIC RESONATOR:
DIAMETER: 12.83MM, HEIGHT: 5.62MM, $\epsilon_r = 38$

Mode	Res. Freq. (GHz, Meas.) [6]	Q_{rad} (Meas.) [6]	Res. Freq. (GHz, Sim.)	Q_{rad} (Sim.)
$TE_{01\delta}$	3.9672	46.4	3.982	40.97
$HEM_{11\delta}$	5.1800	30.3	5.256 (2 modes)	30.55
$HEM_{12\delta}$	5.4032	43.3	5.446 (2 modes)	44.75
$TM_{01\delta}$	6.1328	58.1	6.289	65.75

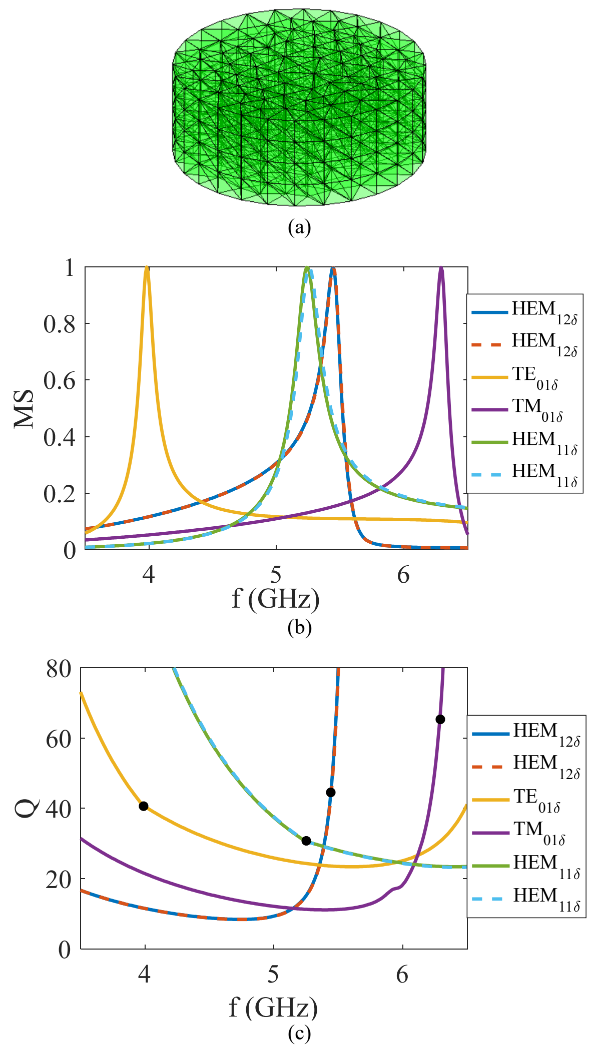

Fig. 1. Simulation results of the cylindrical resonator: (a) geometry in mesh, (b) modal significance, (c) broadband modal Qs (the marks represent the modal resonant frequencies).

References

[1] A. W. Glisson, D. Kajfez and J. James, "Evaluation of Modes in Dielectric Resonators Using a Surface Integral Equation Formulation," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 31, no. 12, pp. 1023- 1029, Dec. 1983.

[2] R. Harrington, J. Mautz and Yu Chang, "Characteristic modes for dielectric and magnetic bodies," in *IEEE Transactions on Antennas and Propagation*, vol. 20, no. 2, pp. 194-198, Mar 1972.

[3] G. A. E. Vandenbosch, "Reactive Energies, Impedance, and Q Factor of Radiating Structures," in *IEEE Transactions on Antennas and Propagation*, vol. 58, no. 4, pp. 1112-1127, April 2010.

[4] H. Alroughani, J. L. T. Ethier and D. A. McNamara, "Observations on computational outcomes for the characteristic modes of dielectric objects," *2014 IEEE Antennas and Propagation Society International Symposium (APSURSI)*, Memphis, TN, 2014, pp. 844-845.

[5] D. Schaubert, D. Wilton and A. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," in *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 1, pp. 77-85, Jan 1984.

[6] R. K. Mongia, C. L. Larose, S. R. Mishra and P. Bhartia, "Accurate measurement of Q-factors of isolated dielectric resonators," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 42, no. 8, pp. 1463-1467, Aug 1994.