Parallel Computation in Hierarchically Semiseperable Methods for Surface Integral Equations

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Recently in the numerical solution of electromagnetic scattering there has been much interest on low-rank matrix representations. In scattering simulations, parts of the method of moments/surface integral equation (MoM-SIE) system matrix may often be approximated as a rank-deficient matrix block instead, with minimum loss in accuracy. This rank-structured property may be exploited in order to solve matrix equations in quicker time, with much smaller memory requirements. One such example in recent years is the adaptive cross approximation algorithm (ACA). Recently, the concept of hierarchically semiseperable (HSS) structures has been developed and applied to compress and solve systems of equations arising in MoM-SIE discretization of scattering problems.

This paper discusses hierarchically semiseperable matrices and their application to MoM-SIE equations – including HSS compression techniques, ULV-type factorization algorithms, and corresponding system solutions. In particular, the paper focuses on the exploitable parallelism present in HSS tree-type structures and their corresponding operations. There are many considerations that have to be taken into account during parallelization – care must be taken when assembling the MoM-SIE system carefully. Proper ordering of the system unknowns affects the rank structure of the system matrix and the degree of compressibility.

A modified sorting technique based on the cobblestone sorting technique is used to order the unknowns in such a way as to exploit the hierarchical and compressible nature of the system matrix. The SIE mesh is boxed and projected along the diagonal of the box to determine a patch around which the remaining SIE unknowns. Unknowns are sorted based on their difference from the reference patch, and the process is repeated multiple times to determine an effective grouping for good parallel performance and system matrix compressibility. This is implemented in a variety of ways in a newly modified hierarchical fashion, and a comparison is made between various sorting techniques.

Effective communication patterns are one of the keys to successful parallelism, and such patterns are important in all phases of the algorithm, including matrix filling, HSS compression, the ULV-type factorization of the corresponding matrix, and the final system back substitution/solution. During all algorithms, sets of processes are combined and split hierarchically into contexts based on the compression structure of the matrix. We discuss utilization and interfacing of the ScaLAPACK and BLACS libraries for optimizing parallel efficiency.

Numerical examples of parallel performance and accuracy of HSS matrices applied to MoM-SIE equations are presented. We observe great accuracy in agreement with direct matrix solving based on Gaussian elimination or LU decomposition, and near-linear parallel performance in speedup in select test cases.