Synthesizing Thin Dielectric Lenses for Conical Scanning Beams: A Hybrid Numerical Algorithm

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***Abstract*—A numerical recipe is presented to synthesize off-axis fed lenses to produce conical beams. The algorithm is based on the concepts of Geometrical Optics Ray Tracing and Particle Swarm Optimization of a 10-term Quadratic-Tapered Legendre Series Expansion of the lens surface topology parameterized as a Body-of-Revolution to obtain azimuthal symmetry required to produce the conical beam. Full-Wave analysis is used to validate the technique. An example of an on-axis fed shaped lens optimized for uniform phase is presented.**

I. Introduction

A particular space borne application arises in which an alternative to a parabolic reflector antenna which produces a conical beam via a motorized scanning mechanism warrants an electronic alternative. One alternative is a spherical Luneburg Lens Antenna [1]. This antenna provides an electronic scan mechanism with performance similar to the motorized scan of the parabolic reflector. However, due to the spherical geometry of the Luneburg Lens, the weight becomes impractical. Thus, a design which reduces the weight of the Luneburg Lens while maintaining similar conical scan and pattern performance must be developed. This paper presents a hybrid Geometrical Optics ray tracing, Particle Swarm Optimization, and Full-Wave Analysis numerical algorithm to design low-weight, azimuthally symmetric, shaped and optimized, dielectric lens antennas to produce scanning conical beams, and thus are a viable candidate to replace the motorized scan of the parabolic reflector antenna.

II. Lens Synthesis, Analysis, and Design Procedure

The hybrid design scheme and numerical algorithm adopted is shown in Fig. 1.

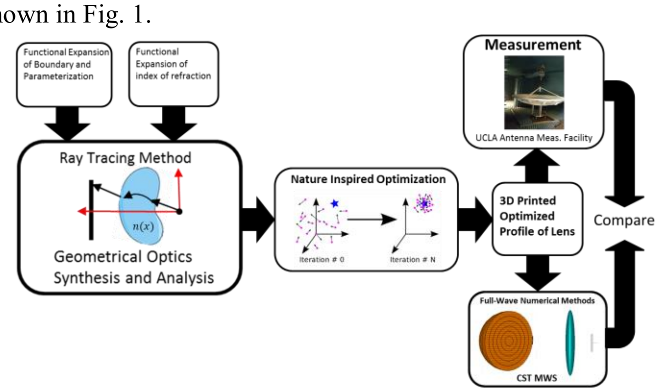

Fig. 1. Lens Design Procedure Flowchart.

The surface of the lens is parameterized as a Body of Revolution (BOR), of which a generating curve is revolved about the lens axis of revolution. In order for the lens to obtain a unique shape for both the upper and lower hemispheres, a different expansion is performed for each hemisphere. The expansion takes the following form

$$f_{upper}(\rho) = +\left(a_0\sqrt{1-\left(\frac{\rho}{a}\right)^2} + \sum_{n=1}^{9} a_n P_{2n}\left(\frac{\rho}{a}\right)\left(1-\left(\frac{\rho}{a}\right)^p\right) \right)^{1/p} \quad (1)$$

$$f_{lower}(\rho) = -\left(b_0\sqrt{1-\left(\frac{\rho}{a}\right)^2} + \sum_{n=1}^{9} b_n P_{2n}\left(\frac{\rho}{a}\right)\left(1-\left(\frac{\rho}{a}\right)^p\right) \right)^{1/p} \quad (2)$$

where $P_n(x)$ are the Legendre Functions, and thus the expansions above can be seen as a quadratic-tapered Legendre-Series Expansion. As to produce a generating curve from the entire expansion, only one half of the expansions above are kept, those for positive ρ. The expansion functions are chosen to be even ordered functions only as to force a zero tangent as $\rho \to 0$ such that the body of revolution is smooth about the revolution axis. The taper is added for two reasons. First in order to force the expansion functions to go to zero at the boundary of the lens (normally the Legendre Functions attain a value of +1 at the ends of their domain of definition) and second to force an infinite tangent as $\rho \to a$ at the rim of the lens such that the upper and lower surfaces join smoothly. The zeroth order or D.C. term is chosen as the equation of an ellipse with an upper and lower minor semi-axis of (a_0, b_0). The additional terms are tapered with a coefficient of p=1.4 in order to allow the lens to approach that of the D.C. ellipsoid near the rim of the lens so that the perimeter of the lens is smooth. Although the orthogonality of the Legendre functions is effected by the addition of the quadratic terms, the expansion functions maintain linear independence. The orthogonality is typically exploited when an analytic expression for the expansion coefficients are sought, however, in this case, the coefficients are chosen by an optimizer and thus an analytic result is unnecessary. A 10-term expansion was found to be suitable for this application. Once the generating curve is produced via (1) and (2), the upper and lower surfaces are parameterized as a BOR using

$$\vec{r}(\rho,\phi) = [\rho\cos(\phi), \rho\sin(\phi), z_c + f(\rho)] \quad (3)$$

and joined to create the surface profile of the shaped lens. z_c defines the geometric center of the lens and f(ρ) is either the upper or lower surface parameterizations of (1) and (2). Launched rays are traced to the aperture using the concepts of Geometrical Optics. In addition to the surface parameterization as given in (3), each segment of the ray and the aperture plane must be parameterized as

$$\vec{r_1}(t_1) = t_1 \bullet \hat{r}_{inc} + \vec{r_f}, \quad \vec{r_2}(t_2) = t_2 \bullet \hat{r}_{trans} + \vec{r_1}(t_1)$$
$$\vec{r_3}(t_3) = t_3 \bullet \hat{r}_{out} + \vec{r_2}(t_2), \quad \vec{r_p}(l,m) = \vec{r_c} + l \bullet \hat{r}_{p1} + m \bullet \hat{r}_{p2} \tag{4}$$

where $t_1, t_2, t_3, l, m \in \mathbb{R}$ and $\vec{r_f}$ points to the feed phase center. Each of (3) and (4) are shown indicated in Fig. 2.

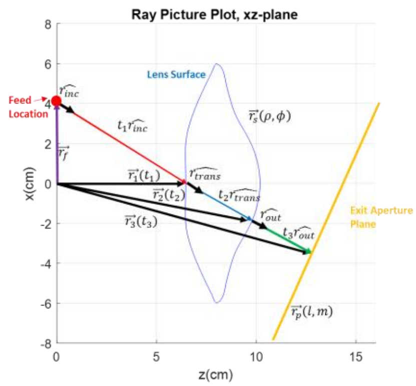

Fig. 2. Ray Tracing Parameterizations.

Once all the parameterizations are defined, the point of intersection between each set must be found. This leads to a 3x3 system of equations for each point of intersection (POI), one equation for each vector component, and since the POI's are always a line and a surface, one variable for the line and two for the surface. The system of equations is solved by Gauss-Newton Root Finding. This algorithm is sensitive to the initial guess. A technique to obtain an accurate initial guess was developed. To find $\hat{r}_{trans}$ and $\hat{r}_{out}$, one applies Snell's law in a local coordinate system at the POI constructed via the local surface tangents obtained via the partial derivatives of the surface parameterization of (3) and their cross-product which gives the local surface normal vector. In order to expedite the computations, the first system to be was reduced to a 2x2 system via coordinate rotations of the surface parameterization of (3). Once all the rays are traced to the aperture, phase-only synthesis is performed via the computation of the standard deviation of the phase in the aperture plane. The 10-term expansion coefficients for both the upper and lower surface generating curve is entered into a Particle Swarm Optimization routine [2] in order to obtain the optimized surface geometry for the shaped lens. The optimizer evaluates each design assigning a scalar fitness value according to the fitness function below

$$fitness = StdDev + 3*(1 - RayRatio) + 0.2*VolumeRatio \tag{5}$$

where StdDev is the standard deviation of the phase in the aperture plane, RayRatio is a figure of merit characterizing the ratio of rays which enter the exit aperture without total internally reflecting while exiting the lens, and VolumeRatio characterizes the reduction in volume of the optimized lens. The optimizer is seeded with the analytic 10-term Legendre Series expansion of (1) and (2) for an Elliptic Curve function as the lens was expected to be of a 'red blood cell' type shape.

III. Results

As an example, a shaped on-axis fed dielectric lens antenna was designed and compared to an ellipsoidal dielectric lens baseline. Both lenses were fabricated using 3D printing technologies and the results of the measurements will be presented. The feed was linearly polarized rectangular microstrip patch located 8cm away from the geometric center of the lens and made to resonate at 13.4GHz. The lens diameter is 12cm. The numerical results are shown in Fig. 3 below

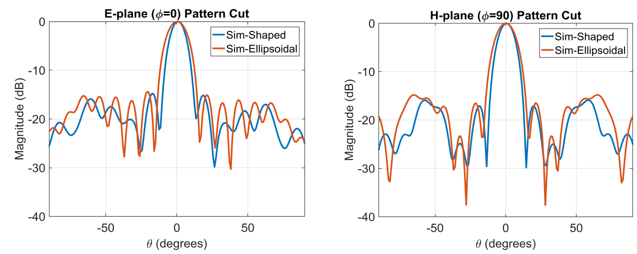

Fig. 3. Shaped vs. Ellipsoidal Dielectric Lens Antenna Far Field Radiation Patterns.

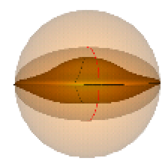
Fig. 4. Cross-Sectional View of Equivalent Luneburg, Ellipsoidal, and Shaped Dielectric Lens.

The peak directivity of the shaped lens is 20.51 dB, a 1.5 dB improvement over that of the ellipsoidal baseline. Finally, note that the volume was reduced by nearly 80% relative to the Luneburg equivalent as pictorially shown in Fig. 4.

IV. Conclusion

A hybrid numerical recipe is given to synthesize lenses for conical beams using the concepts of Geometrical Optics Ray Tracing and Particle Swarm Optimization. An example of an on-axis fed shaped dielectric lens antenna was given and the results compared to an ellipsoidal baseline. Significant improvements were seen in performance and weight.

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References

[1] R. Luneburg and M. Herzberger, Mathematical Theory of Optics, University of California Press, 1964.

[2] J. Robinson and Y. Rahmat-Samii, "Particle Swarm Optimization in Electromagnetics," IEEE Trans. Antennas and Propagation, Vol. 52, No. 2, pp. 397–407, Feb. 2004.