

## **Modeling VLF Transmitter Amplitude and Phase Variations in the Earth-Ionosphere Waveguide**

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Very-low-frequency (VLF, 3—30 kHz) waves can propagate efficiently in the waveguide formed by the Earth and the D-region ionosphere. Variation in the signals monitored by a stationary receiver can be attributed to variations in the lower ionosphere. As such, these signals are used to monitor the D-region ionosphere in daytime and nighttime. However, the use of VLF transmitter signals to quantitatively diagnose the D-region ionosphere is complicated by i) the propagation of many modes in the waveguide, and their interference, and ii) the effect of the ionosphere along the entire path on the receiver signal at a single location.

In this paper, we compare the modeled phase and amplitude of VLF signals using three methods: a Finite-Difference Time-Domain (FDTD) model, a Finite-Difference Frequency-Domain (FDFD) model, and the Long-Wave Prediction Capability (LWPC) model, which has been the method de rigueur since the 1970s. While LWPC solves mode propagation and coupling in the anisotropic waveguide, the FD methods directly solve for electric and magnetic fields from Maxwell's equations on a finite-difference grid. Thus, FD methods provide greater freedom to vary the physical inputs of the model, limited only by the spatial resolution, but at the expense of computation time.

We compare the simulated amplitude and phase of these models by running them with identical physical inputs. In this work we compare both i) the absolute amplitude and phase trends as a function of distance, and ii) the magnitude of amplitude and phase variations for given ionosphere changes. Modeling results show that FDTD and FDFD simulations track the amplitude and phase as a function of distance very closely when compared to LWPC. Phase drift due to numerical dispersion is observed at large distances, of a few tens of degrees per 1000 km. These phase drifts increase quadratically with frequency, as expected from numerical dispersion in FD methods. In fact, the phase drift can be mostly removed by applying a simple Richardson extrapolation. After extrapolating, FDTD and LWPC differences can be mapped to a phase velocity difference of  $<0.07\%$ . When we compare phase changes due to ionospheric variations, we find that all three models show similar magnitudes of phase changes, to within  $\sim 20\%$ , and similar trends with frequency.