A Comparison of Integration Schemes for Sommerfeld Integral Evaluation in the Half-Space Problem

Dawei Li, Donald. R. Wilton, David. R. Jackson and Ji Chen
Department of ECE, University of Houston, Houston, TX 77204-4005

Since Sommerfeld first conducted a detailed analysis of the radiation problem for an infinitesimal vertical Hertzian dipole over a lossy medium in 1909, the classical half-space problem has received continuous attention. The dipole solution (Green's function) for the half-space problem is expressed in terms of the Fourier-Bessel transforms known as Sommerfeld integrals. These integrals in the wavenumber plane do not have closed forms in general, and evaluation of these integrals is challenging for various reasons. There are branch cuts from the branch points corresponding to the wavenumbers of the two unbounded regions, as well as a Zenneck wave pole in the wavenumber plane. Furthermore, the integrand is slowly decaying and rapidly oscillating on the real axis. In this work a comparison of two different techniques for the evaluation of Sommerfeld integrals in the half-space problem that arise in the mixed-potential formulation are studied.

The double-exponential (DE) algorithm with the trapezoidal rule is explored on several possible paths in the complex plane. The DE method is very effective in integrating any smooth, convergent integrand whose singularities occur only at the path endpoints. When using the DE integration scheme, high accuracy is achieved by using the simple trapezoidal or the midpoint rule of integration, since the integrand is smooth and has infinite limits. One path that can be chosen goes from the origin in the wavenumber plane straight to the nearest branch point, then directly to the other branch point, finally returning to the real axis. Once on the real axis, the weighted average (WA) method can be used to efficiently evaluate the tail. Asymptotic extracted forms used to accelerate the spectral integrals introduce a spurious pole, resulting in the divergence of a few of the integrals. An extracted "pole function" is introduced to cancel the singularity, while its contribution is accounted for analytically.

When the distance between source and observation points increases, the integrands oscillate faster due to the Bessel functions, resulting in a low integration efficiency. In this case, a steepest-descent path (SDP) can be used to accelerate the convergence (K. A. Michalski and C. M. Butler, Evaluation of Sommerfeld integrals arising in the ground stake antenna problem, IEE Proc., Vol. 134, Pt. H, No. 1, pp. 93-97, Feb. 1987.) The nature of the SDP and the convergence of the integrals are examined for cases where the source and observation points are in the same region, and when they are in different regions. Care must be taken to treat the branch cut that remains after employing the steepest-descent transformation, which removes only one of the two branch cuts in the wavenumber plane.