Examination of the Near Field Response of Circular Antenna Arrays

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***Abstract*— A uniformly distributed family of circular topology is analyzed using probabilistic methods to find mean valued radiation characteristics in the near field (Fresnel zone). Linear arrays with circular and spherical distributions of one, two and three dimensional spaces are compared using this analysis to obtain benchmark performance metrics. This novel method also determines the statistical effects of converging pattern behavior at the hyperfocal distance of each topology. Closed form expressions from this method provide ease in statistical analysis. As a result, this work presents a comparison of data for each respective circularly bounded distribution with its respective hyperfocal distance. Results of each dimensional topology also maintain the same aperture size in comparison to each respective distribution range resolution capability.**

***Index Terms*—Collaborative beamforming, distributed beamforming, and wireless sensor networks.**

I. Introduction

Random antenna arrays have shown superior performance characteristics when compared to their periodic counterparts [1] – [3]. The mathematical derivation of this for a spherically bound random antenna array has been shown in previous papers [1] – [4] for determining these characteristics. The implementation cost is reduced since fewer elements are used. Also many known issues with traditional phased arrays are avoided and considered neglible for the random array. These include impedance mismatch, mutual coupling effects, blind scan angles and narrow bandwidths. In addition, arrays of spherical symmetry tend to outperform traditional linear or planar types. For instance, linear and planar arrays under even the most ideal conditions will have radiation characteristics that tend to deteriorate rapidly for in-plane beamsteering [1]. This is not seen with spherical symmetry. For this reason spherical geometry is more attractive [1] – [3]. This work highlights the behavior of spherical topology as well as other circularly symmetric topologies such as planar (circular) and linear distributions of equivalent aperture size.

First a mathematical explanation of the mean valued radiation pattern is shown for the spherical random array (SRA), circular random array (CRA) and linear random array (LRA). Results of this derivation are critical for relating to the historical SRA works in [4]. In effect, this allows for ease in the calculation of frequency and hyperfocal distance. An examination of the hyperfocal distance of the LRA, CRA and SRA is done first, followed by a comparison of one result. A short discussion follows.

A. Expected Value Radiation Pattern

It has been shown from [1] that taking the expected value of the array factor across the unit interval [-1, 1] for a random variable x in any volumetric distribution provides a general theory towards obtaining the characteristic functions of (1), which are orthogonal in all three axis and uncorrelated.

$$\bar{U} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\Lambda|u|^2\Lambda|v|^2\Lambda|w|^2 = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\Lambda\left|\vec{\Psi}\right|^2,$$
$$\vec{\Psi} = \cos^{-1}\left(kA\left(\hat{\mathrm{r}}(\theta,\phi) - \hat{\mathrm{r}}(\theta_0,\phi_0)\right)\right) = \cos^{-1}\left[u \quad v \quad w\right] \tag{1}$$
$$u = \hat{\mathrm{x}}\cdot\cos\vec{\Psi}, v = \hat{y}\cdot\cos\vec{\Psi}, w = \hat{z}\cdot\cos\vec{\Psi}, 1 = \sqrt{u^2+v^2+w^2}$$

In other words, [1] has determined that the main beam and first sidelobe level of the characteristic function have fairly deterministic properties. As a consequence, the same methods used in aperture theory may be applied to distributed arrays [1]. To analyze this methodology a simplified expected pattern is used from (1) in terms of *Psi space* (Ψ) where Ψ is taken over all angular space (θ, ϕ), similar to the u, v and w space of traditional array theory [1]. Results of the near field characteristic function can be simplified with the introduction of the parameter t as provided in (10). By doing so the LRA, CRA and SRA near field solution can be calculated, see (11-13) respectively.

II. Power Pattern in the Fresnel Region

A. Array Factor with Directional Sine Coefficient

A near field derivation of the array factor is derived in this section for a uniformly distributed volumetric random array with perfect phase information bound to a Euclidean domain. It begins with describing the element distance R_n (θ, ϕ) in (2) as the Euclidean distance between P_n and P with its first four expansions provided by that of (3) - (6).

$$R_n = |r - r_n| = r\sqrt{1 + x^2 - 2x\cos\psi_n},\ x \triangleq \left(r_n/r\right)$$
$$R_n \approx \sum_{n=0}^{N} r\binom{1/2}{n}\left(x^2 - 2x\cos\psi_n\right)^n,\ \sin\psi_n = \sqrt{1-\cos\psi_n^2} \tag{2}$$
$$\cos\psi_n = \hat{r}_n\cdot\hat{r} = \sin\theta_n\sin\theta\cos\left(\phi-\phi_n\right) + \cos\theta_n\cos\theta$$

$$R_n\big|_{N=0} \approx r \tag{3}$$

$$R_n\big|_{N=1} \approx r - r_n\cos\psi_n + r_n^2/2r \tag{4}$$

$$R_n\big|_{N=2} \approx r - r_n\cos\psi_n + r_n^2\sin^2\psi_n/2r \tag{5}$$

$$R_n\big|_{N=3} \approx R_n\big|_{N=2} + \left(\frac{r_n^3}{2}\cos\psi_n \sin^2\psi_n\right)\Big/ r^2 + \ldots \qquad (6)$$

When P is assumed to reside in the near field (Fresnel zone) of the array, common practice involves a first order approximation for the magnitude (3) and a third order approximation for the phase (4). In fact, (3) - (4) are the common far field approximations where the hyperfocal range term (r^2_n /2) is often neglected. This region occurs where the largest dimension of the array satisfies $8A^2 > \lambda$, where A is the radius of the aperture (note: this is different from the typical notation of $D = 2A$; used in this work for circularly symmetric topologies). Here, a maximum phase error of 22.5° is achieved and considered acceptable, but decreases the overall gain by 3dB. This can be critical and perhaps detrimental to collaborative beamforming applications, especially when trying to beamform close to a sparse array (i.e. in the near-field). Hence, if one is to investigate a non-negligible third order approximation the array factor becomes that of (7) using that of (5) for a third order approximation where the directional cosine ($\cos\psi_{n0}$) and sine ($\sin\psi_{n0}$) terms of the n^{th} element provide for beamsteering capability at the direction P_0 and hyperfocal range r_h along with uniform amplitude excitation. Lastly, higher order moments of the array factor can be found similarly to that of (7) by applying that of (6).

$$F(r,\theta,\phi|r_n,\theta_n,\phi_n) \approx \frac{1}{N}\sum_{n=1}^{N} e^{jkA\left(r_n(\cos\psi_n - \cos\psi_{no}) + r_n^2 A\left(\frac{\sin^2\psi_n}{2r} - \frac{\sin^2\psi_{no}}{2r_H}\right)\right)}$$

$$F(r,\theta,\phi|r_n,\theta_n,\phi_n) \approx \sum_{n=1}^{N} e^{jkA\left(r_n(u) - r_n^2 A\left(\frac{uv}{2r_H}\right)\right)} \Big/ N \Bigg|_{r=r_h} \qquad (7)$$

$$u = (\cos\psi_n - \cos\psi_{n0}),\ v = (\cos\psi_n + \cos\psi_{n0})$$

B. Array Factor without Directional Sine Coefficient

In the Fresnel region simplification of the traditionally studied array factor [5] takes the form in (8) and in the form of (9) for the traditional beamsteering point ($\psi_n = \psi_{n0}$).

$$F(r,\theta,\phi|r_n,\theta_n,\phi_n) \approx \sum_{n=1}^{N} e^{jkA\left(r_n(\cos\psi_n - \cos\psi_{no}) + r_n^2 A\left(\frac{1}{2r} - \frac{1}{2r_H}\right)\right)} \Big/ N \qquad (8)$$

$$F(r,\theta_0,\phi_0|r_n,\theta_n,\phi_n) \approx \sum_{n=1}^{N} e^{jkA\left(r_n^2 A\left(\frac{1}{2r} - \frac{1}{2r_H}\right)\right)} \Big/ N \qquad (9)$$

To solve for the mean valued radiation pattern one uses the parameter t as shown in (10) and providing solutions (11) – (13) for the LRA, CRA and SRA respectively.

$$\Lambda(t|\Psi=0) = |F(t|\Psi=0)|^2 = \int_{-1}^{1} e^{itx^2} f_x(X)\,dx$$
$$t^2 = A^2\pi/\lambda(1/r_h - 1/r_1) = A^2\pi/\lambda(1/r_h)\Big|_{r_1=\infty} \qquad (10)$$

$$\Lambda(t) \overset{LRA}{=} -(-1)^{1/4}\sqrt{\pi}\,Erf\left[(-1)^{3/4}\sqrt{t}\right]\Big/2\sqrt{t} \qquad (11)$$

$$\Lambda(t) \overset{CRA}{=} \exp(jt/2)\left(J_0(t/2) - jJ_1(t/2)\right) \qquad (12)$$

$$\Lambda(t) \overset{SRA}{=} 3je^{jt}\left(j\sqrt{jt} + (-j+2t)\mathrm{D}\left(\sqrt{jt}\right)\right)\Big/4(jt)^{3/2}$$
$$\mathrm{D}(x) = e^{-x^2}\int_0^x e^{-t^2}dt \ \ (\text{Dawson Function}) \qquad (13)$$

Graphical representations of (11) – (13) are provided in Fig.1 and are useful when solving for r_h. The illustration resembles the traditional radiation pattern; however, the 3dB point of the graphical solution is only necessary to find r_h. Results of the LRA, CRA and SRA hyperfocal distances are also provided in Table 1 for the one (linear), two (circular) and three (spherical) dimensional random arrays for completeness.

Table 1. Hyperfocal Distance Comparisons.

Array/focus	SRA	CRA	LRA
r_h	$.77A^2/\lambda$	$.93A^2/\lambda$	$1.15A^2/\lambda$
r_1	$.39A^2/\lambda$	$.47A^2/\lambda$	$.58A^2/\lambda$

A comparison of the hyperfocal distances of the linear random array matches classical solutions [5], though the solution of the spherical random array given in [4] differs ($r_h = r^2/.9855\lambda$). This is due to a miscalculation of an integral representation and has been corrected in Table 1.

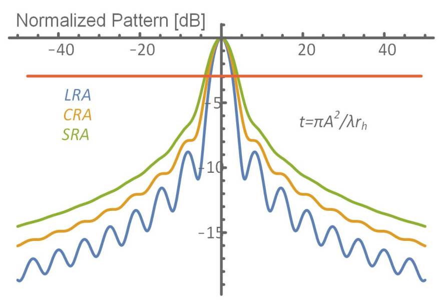

Fig. 1. Near-field solutions conditioned at $r_2=\infty$.

III. CONCULSION

This work calculated the hyperfocal distance capabilities of a LRA, CRA and SRA. Of the one, two and three dimensional topologies studied, the SRA has the closest hyperfocal distance to the array and the LRA has the furthest.

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