Boundary Conditions for Multipolar Media Determined from Maxwell's Equations and Constitutive Relations

Arthur D. Yaghjian
Electromagnetics Research Consultant, Concord, MA, 01742,
a.yaghjian@comcast.net

The usual textbook derivation of the boundary conditions across the interface between two different dipolar media with finite conductivity finds that the tangential components of **E** and **H** are continuous across the interface. However, extreme values of constitutive parameters equal to zero or infinity can produce delta functions in the polarization densities **P** and **M** at interfaces that imply discontinuous tangential components of **E** and **H** across the interfaces (A.D. Yaghjian, Metamaterials, 4, 70-76, 2010). Moreover, spatially dispersive constitutive parameters can also produce discontinuities in the tangential components of **E** and **H** across interfaces (L.D. Landau et al., Electrodynamics of Continuous Media, 2nd Edition, Oxford, UK: Butterworth Heinemann, p. 361, 1984.) Therefore, it is evident that boundary conditions depend not only on Maxwell's equations but also on the particular constitutive relations satisfied by the material on either side of the interface. Since the constitutive relations also determine the number of eigenmodes that exist in a medium at any one frequency, and the number of eigenmodes determine the number of boundary conditions required to uniquely solve an interface boundary value problem, the number of independent boundary conditions depends on the particular constitutive relations.

Allowing the polarizations and the fields to have delta functions in the transition layer of interfaces while noting that the constitutive parameters must remain finite everywhere, a unique set of a deterministic number of interface boundary conditions can be derived from Maxwell's differential equations. We illustrate the technique by deriving three interface boundary conditions (two conventional and one "additional boundary condition" [ABC]) for electric quadrupolar media satisfying a physically reasonable constitutive relation that supports both an evanescent and a propagating eigenmode. Numerical computations show that for long wavelengths, two previous boundary conditions, derived under the assumption that the electric quadrupolarization contains negligible effective delta functions in the transition layer, produce an accurate solution by neglecting the evanescent eigenmode, that is, by assuming it decays within the transition layer. It appears that this general method used to derive the electric quadrupolar boundary conditions, including the ABC, can be applied to obtain the boundary conditions for any other realizable constitutive relation in a Maxwellian multipole continuum.