Seawater Dielectric Measurement by using a Cavity Technique: Exit-hole Effect Analysis

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Abstract — **The resonant cavity technique is considered as one of the most accurate methods for determining dielectric constants of liquids. Using this technique, the sample is introduced into the cavity through a hole in a glass tube centered on the cavity's axis. The glass tube enters and leaves the cavity through a small hole in each end plate. These exit-holes have an effect on the electromagnetic fields inside the cavity and change its resonant frequency. The effect of these exit-holes has concerned investigators for years. In this paper, the exit-hole effect has been analyzed using waveguide theory. Higher order modes have been taken into account to increase the accuracy of the method. In addition, a special experimental setup has been designed to validate this theory.**

Key words **— seawater; permittivity; exit-hole effect; waveguide theory**

I.INTRODUCTION

The cavity technique is often used for determining the dielectric constant of liquids. A glass tube is inserted into the cylindrical cavity along its axis through a small hole in the top and bottom plates. The glass tube contains a small hole along its axis through which the liquid is introduced into the cavity. The subject of this paper is the effect of the holes in the cavity end plates on the measurement of the liquid's dielectric constant. The investigation of the so called exit-hole effect is based on the experimental setup of the seawater dielectric measurements at The George Washington University.

In this experiment, a transmission type cavity technique has been used for determining the seawater dielectric constant at L-band (1.413 GHz). In the measurement system, the seawater is introduced into a brass microwave cavity through a capillary glass tube having an inner diameter of 0.1 mm. After introducing the seawater sample into the cavity, the seawater perturbs the field inside the cavity causing a change in both the resonant frequency and the cavity Q. Under the assumption that the amount of seawater introduced into the cavity is small, the complex permittivity of seawater can be retrieved by using the following perturbation relations:

$$\varepsilon' - 1 = 2C\Delta f / f_o \quad , \quad \Delta f = f_o - f \tag{1a}$$

$$\varepsilon'' = C\Delta(1/Q) \quad , \quad \Delta(1/Q) = 1/Q - 1/Q_o \tag{1b}$$

where ε' and ε'' are the real and imaginary parts of the relative dielectric constant of the seawater sample and C is a calibration constant. The variables f_0, Q_0 and f, Q are the resonant frequency and the quality factor of the cavity before and after the sample solution has been introduced respectively. In the perturbation formulas, the calibration constant C has been determined by using a reference liquid with a known dielectric constant. In this work the measurements made by Gregory and Clark [1] of the dielectric constant for methanol at 20°C are employed as the reference liquid to determine the calibration constant C. The details of these seawater dielectric measurements are documented in Lang et. al. [2].

II. EXIT-HOLE EFFECT ANALYSIS

The effect of these exit-holes has concerned investigators for years. Ho and Hall [3] used an approximate method derived by Estin and Bussey [4] showing that the error introduced in the measurement of the dielectric constant was less than 0.1% for a cavity at 1.43 GHz. However, in the calculation provided by Estin and Bussey, only TM_{010} mode is considered in the tube region. More recently, Risman and Wappling-Raaholt [5] have pointed out that the exit-holes might excite a quasi-TEM wave in the hole region. Their computer simulation showed a much larger frequency shift due to the exit-hole effect.

Recently, The George Washington University started investigating the effect of the exit-hole. Due to the complexity of the exit-hole problem of the cavity with capillary tube and sample solution, an analysis of the empty cavity with a small hole has been performed. In the experimental set-up, a brass plunger has been installed right above the exit-hole region. The plunger is sealed to a 40-mm bolt and the depth of exit hole can be adjusted by

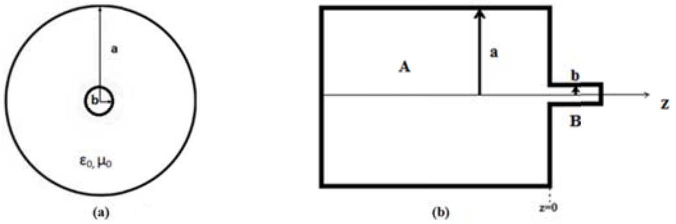

Figure 1 Empty cavity with exit hole geometry [(a) transverse plane (b) side view]

turning the bolt, which results the resonant frequency changing simultaneously. This measurement shows that the existence of exit-hole increases the frequency by about 4 kHz.

The exit-hole effect has also been validated by performing a theoretical analysis. This problem can be viewed as different sections of a step-cylindrical waveguide. Figure 1 shows the transverse and longitudinal planes of the empty cavity with exit-hole under consideration. In the Figure 1(b), the empty cavity with radius *a* is represented by region A while the exit-hole with radius *b* is represented by region B. In the empty cavity (region A) only the dominant mode (TM_{010}) can propagate while the higher order modes are cut-off. All the modes are cut-off in the exit-hole region. Thus, the electric fields in both regions can be written as,

$$E_{\rho A} = \mathbf{e}_{A1}(\rho)\cdot(e^{-j\kappa_{A1}z} + \Gamma_1 e^{+j\kappa_{A1}z}) + \sum_{m=2}^{\infty} \mathbf{e}_{Am}(\rho)\cdot\Gamma_m e^{+\alpha_{Am}z}, \quad z \le 0 \quad (2a)$$

$$E_{\rho B} = \sum_{n=1}^{\infty} \mathbf{e}_{Bn}(\rho) T_n e^{-\alpha_{Bn}z}, \quad z \ge 0 \quad (2b)$$

where $\mathbf{e}_{Am}$ is the mode function in region A with the order of *m*, $\mathbf{e}_{Bn}$ is the mode function in region B with the order of *n*. Γ_m is the reflection coefficient due to the exit-hole effect. κ_{A1}, α_{Am} are the propagation constant of the dominant mode and the attenuation constant of the *m*th order mode in region A, respectively. α_{Bn} is the attenuation constant of the *n*th order mode in region B. By matching the boundary condition at interface z=0, the reflection coefficient Γ_m can be obtained. Note that at z=0, all higher order modes are excited and have to be taken into analysis.

The frequency shift can be determined by using waveguide theory. The cavity is equivalent to a transmission line in z direction as shown in Figure 1(b). The effect of the exit-hole is represented by the first order of the reflection coefficient, Γ_1 , on the end of the transmission line. The frequency shift can be found by solving for the resonant frequency of the equivalent circuit with and without the exit-hole effect.

The difference between the theoretical result and the measurement result is only 1.87%. Comparing to the result provided by Estin and Bussey [4], the difference is about 18%. Therefore, taking the higher order modes into the analysis substantially increases the accuracy of the theoretical model.

III. CONCLUSION AND FUTURE WORK

In summary, the effect of the cavity exit-hole has been calculated accurately by using waveguide theory. A small difference has been observed when comparing the theoretical and experimental results. This difference could be due to the fact that the loss on the wall hasn't been taken into account. In the future, the influence of the cavity lossy wall on the exit-hole effect will be estimated.

In addition, the investigation of the composite exit-hole effect has been started based on this method. In this case, the exit-hole region has multiple layers consisting of the capillary tube wall and sample solution. This analysis consists of three main steps: 1. matching the boundary conditions on the transverse cross-sections of the cavity region and the exit-hole region to determine the eigenvalues, 2. matching the boundary conditions at the interface of the exit-hole to determine the reflection coefficients, 3. determining the frequency shift based on the waveguide theory. Finally, the bias in the real part of the seawater dielectric constant due the exit-hole effect will be obtained. In the future, the calculated result of the composite exit-hole effect will be presented.

IV. REFERENCES

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