# A Simplified Propagation Channel Model for Evaluating MRC Diversity Characteristics in SIMO OFDM with Insufficient Guard Interval

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*Abstract*— We proposed a simplified propagation channel model for evaluating maximal-ratio combining (MRC) diversity characteristics in OFDM where multipath delay profile exceeds the guard interval. The model, we call "Equivalent Transmission-Path (ETP) model", is characterized by a two-wave model which can predict irreducible error rate (BER floor) very accurately. By extending our previously developed ETP model for SISO OFDM system, we develop a model, and the prediction accuracy is identified by comparing with computer simulation results.

Keywords—OFDM, MRC, ETP model, irreducible error, ISI, insufficient guard interval.

### I. INTRODUCTION

In multipath propagation environment where the delay profile exceeds the guard interval (GI) of OFDM, irreducible errors due to inter-symbol interference (ISI) with inter-carrier interference (ICI) cause serious degradation in wideband signal transmission performance. Since it seems difficult to perform a statistical assessment of the irreducible errors theoretically, the transmission performance has been estimated by using computer simulation or partial theoretical approaches without closed form of BER calculation formula.

For this purpose, in our previous study, we have proposed an equivalent transmission-path model (ETP model) using a simple 2-wave model to express a multipath propagation environment with a large delay spreading for single-carrier transmission systems [1], then we have extended the model to make statistical assessments of the characteristics of the irreducible errors in a single-input single-output (SISO) OFDM system [2].

This paper further extends the model from SISO OFDM application to single-input multiple-output (SIMO) OFDM one with maximal-ratio combining diversity and evaluates the accuracy of model by using computer simulation.

# II. ETP MODEL FOR ESTIMATING BERFLOOR FOR SISO IN OFDM

The ETP model in [2] for SISO OFDM application is a simplified channel model for theoretically estimating the transmission characteristics, in which the multipath delay environment is transformed to a 2-wave model, while the values of the key parameters of the radio propagation environment are preserved. The floor value of BER under frequency-selective fading in the ETP model is given by the following equation, taking the amplitude ratio  $r (\equiv |a_{e2}/a_{e1}|)$  and the phase difference  $\phi (\equiv \arg(a_{e2}/a_{e1}))$  of the two waves as variables:

$$BER_{floor} = \iint f_{r\phi}(r,\phi:\Delta\tau_e) P_0(r,\phi:\Delta\tau_e) \, dr d\phi \qquad (1)$$

where  $f_{r\phi}$  is the joint probability density function (PDF) of rand  $\phi$  expressed in the 2-wave model.  $P_0$  is the BER of the 2-wave model as a function of r and  $\phi$ . Since there are no good formulas for calculating the floor value of BER of the 2-wave model,  $P_0$  is used from a table of pre-calculated values which is called "BER map". Figure 1 shows an image of ETP model for irreducible error analysis given in (1).



Fig. 1. Basic concept of ETP model for BER floor analysis.

## III. ETP MODEL FOR SIMO-MRC IN OFDM

# A. Modeling

1) fundamentation

Figure 2 shows how the SIMO channel is transformed into the 2-wave model. The fundamental calculation formula for BER<sub>floor</sub> has much in common with (1). We separate the joint probability density function (PDF)  $f_{r\phi}$  into functions  $f_r$  and  $f_{\phi}$ and attempt to formulate  $f_{r\phi}(r, \phi)$  as  $f_r(r) f_{\phi}(\phi)$ .



Fig. 2. ETP model application for OFDM MRC analysis.

#### 2) PDF of $r : f_r$

The PDF for each amplitude of *n*-th element  $(a_{el,n}, a_{e2,n}: n=1, 2, ..., N)$  of the 2 waves expressed in the ETP model is an independent Rayleigh distribution. The PDF of the amplitude after MRC can be approximated by the Nakagami-m distribution. PDF of the amplitude ratio *r* of two waves can be expressed in the following equation:

$$f_r(r) = \frac{2\Gamma(m_1 + m_2)\alpha^{m_1} r^{2m_2 - 1}}{\Gamma(m_1)\Gamma(m_2)(\alpha + r^2)^{m_1 + m_2}} \qquad \alpha \equiv \frac{\Omega_2 m_1}{\Omega_1 m_2}$$
(2)

where  $m_i$  and  $\Omega_i$  for i=1, 2 are the parameters of the Nakagami-m distribution,  $\Gamma$  is the gamma function.

3) PDF of  $\phi$ :  $f_{\phi}$ 

The PDF of phase variations for each of two waves may be followed by that of Nakagami-m fading. However, the simplified PDF formula is not available yet, we approximate it using the Nakagami-Rice fading given by

$$f_{NR_{-\phi_{i}}}(\phi_{i};K_{i}) = \frac{1}{2\pi} \exp(-K_{i})$$

$$\times \left\{ 1 + \sqrt{\pi K_{i}} \cos \phi_{i} \exp\left(K_{i} \cos^{2} \phi_{i}\right) \left[ 1 + \operatorname{erf}\left(\sqrt{K_{i}} \cos \phi_{i}\right) \right] \right\}$$
(3)

where  $K_i$  is the Rician factor and erf is the error function.

The variance of each phase variation can be estimated by

$$\sigma_{\phi_i}^2 = \left\langle \phi_i^2 \right\rangle = \int_{-\pi}^{\pi} \phi_i^2 f_{NR_{-}\phi_i}(\phi_i; K_i) d\phi \tag{4}$$

Since derivation of PDF for phase difference  $\phi$  based on this equation is still quite complicated, we took further approximation using a truncated Gaussian distribution as follows.  $(\phi^2)$ 

$$f_{\phi}(\phi;\sigma_{\phi}) = \frac{\exp\left(-\frac{r}{2\sigma_{\phi}^{2}}\right)}{\sqrt{2\pi}\sigma_{\phi}\operatorname{erf}\left(\frac{\pi}{\sqrt{2}\sigma_{\phi}}\right)} \quad (-\pi < \phi \le \pi)$$
(5)

where  $\sigma_{\phi} = \sqrt{\beta \left(\sigma_{\phi 1}^2 + \sigma_{\phi 2}^2\right)}$  with  $\beta = 0.65$ .

# B. Evaluation:

Computer simulation was used to evaluate the performance of the proposed method. The exponential delay profile and the multi-cluster delay profile (i.e. Sally-Valenzuela model) were applied for the multipath environment. The results of irreducible errors (BER floor) against delay parameters and OFDM parameters are shown in Fig. 3. As we can see from the figure, the calculation results matched very well with the simulation results. Consequently, the validity of the model is confirmed.





(b) Multi-cluser delay profile

Fig. 3. BER floor characteristics for OFDM/DQPSK: calculated vs. simulated.

#### IV. CONCLUSION

We have developed a simple BER floor calculation scheme for SIMO-MRC OFDM system based on a simple propagation channel model. Due to page limit of this summary, we will give evidences about development and assumption of Eqs. (2) and (5) in the presentation.

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