Subroutines for the computation of radial Mathieu functions for large values of the parameter

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An algorithm for the computation of Mathieu functions for large values of the parameter is presented. This algorithm extends the range of values for which Mathieu functions may be computed with respect to what is described in (Erricolo and G. Carluccio, "Algorithm 934: Fortran 90 subroutines to compute Mathieu functions for complex values of the parameter," *Association for Computing Machinery Transactions on Mathematical Software*, Vol. 40, Issue 1, Sept. 2013). In electromagnetics, Mathieu functions appear as solutions of the Helmholtz equation in the elliptic cylinder coordinate system and find applications in problems where elliptical shapes are involved.

In a typical application, one is interested in the evaluation of angular and/or radial Mathieu functions, for a certain value of the argument, with a certain type of parity (even or odd), for a specific order n and a value s of the parameter (representing the magnitude of the wavevector in electromagnetics problems). In the case of radial Mathieu functions, the *kind* must also be specified. The numerical evaluation of Mathieu functions occurs in three steps (Staff of the Computation Laboratory, *Tables Relating to Mathieu Functions,* 2nd Ed. Applied Mathematics Series. U.S. Government Printing Office, Washington, D.C., 1967). In the first one, starting from n and s, the eigenvalue a_n that satisfies Mathieu differential equation is determined. Mathieu functions are represented as the sum of series so that the second step is the computation of the expansion coefficients D_k of such series, which depend on the parameter s, the order n and the eigenvalue a_n. In the third step, the sum of the series is computed. This computation is quite straightforward for the angular Mathieu functions; however, for the radial Mathieu functions complications may arise when the value of the parameter s is large. We present an approach to overcome this challenge and discuss its numerical application to some case studies.