Scalar Potential Formulation and Depolarizing Dyad Artifact Removal for a Gyrotropic Medium

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Field and vector potential based methods are both employed in the electromagnetic analysis of problems involving simple (i.e., linear, homogeneous, and isotropic) media. Recently, alternative techniques have been developed in order to accommodate complex materials such as anisotropic and bianisotropic media (e.g., I. Lindell, A. Sihvola, K. Suchy, "Six-vector formalism in electromagnetics of bi-anisotropic media", Journ. Electro. Waves and Appl., vol. 9, no. 7/8, pp. 887-903, 1995). This interest has been strongly influenced by the profound developments in material fabrication capability and the added control over scattered fields that complex media can offer. Gyrotropic media, such as biased ferrites, are readily available and find use in many practical devices such as circulators and isolators. Thus, there is precedence in seeking alternative methods capable of handling gyrotropic media, especially formulations that can ease mathematical effort and enhance physical insight.

The goals of this paper are to first develop a scalar potential formulation for a magnetically and electrically gyrotropic anisotropic medium. It is shown that two primary and two secondary scalar potentials arise in the development. The physical relationship between these potentials and corresponding electromagnetic fields is discussed. Next, depolarizing dyad terms arising in the development are identified, one that is expected and another that is unexpected. It is discussed, from a physical viewpoint, why the unexpected depolarizing dyad term should not exist. Based on this insight, the final goal is to mathematically demonstrate the unexpected depolarizing dyad is actually removable.

The removal of the unexpected depolarizing dyad is demonstrated for a gyrotropic medium by first Fourier transforming the scalar potential wave equations from the spatial domain to the spectral domain to readily solve for the two primary scalar potentials. Subsequent Fourier inversion, with the aid of complex plane analysis, leads to spatial domain expressions for these potentials. Next, the two secondary potentials, which involve spatial derivatives, are found from the two primary potentials. Through careful handling of the spatial derivatives via Leibnitz's rule, it is demonstrated the unexpected depolarizing dyad term encountered in the scalar potential development is actually removable. This demonstration is crucial in order to obtain a physically and mathematically consistent and useful theory.

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