Scattering anomalies for radially anisotropic spheres

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The scattering behavior of electrically small particles has a very strong frequency dependence: according to the so-called Rayleigh scattering law, the scattering cross section increases proportionally to the fourth power of the size-to-wavelength ratio. For spherical scatterers, this law can be quantized from the long-wavelength limiting behavior of the Lorenz–Mie scattering coefficients. For small dielectric inclusions, in addition to the fourth-power dependence on electrical size, the scattering efficiency is proportional to the square of the absolute value of the static polarizability. However, as is known, the polarizability function experiences a singularity, which for a sphere takes place for the relative permittivity value of –2 (corresponding to lossless plasmonic material response). Around this singularity, the fourth-power Rayleigh dependence fails. For a purely dielectric particle, the scattering efficiency is constant with increasing electrical size of the sphere. An even more astounding result is that if the sphere has a particular magnetic response (relative permeability being –5), the size dependence power is –2 (instead of +4 like in the Rayleigh case).

The diversity of scattering anomalies becomes much richer when the scatterer is allowed to be anisotropic. In particular, so-called radially anisotropic (RA) spheres display very interesting scattering and extinction characteristics (Wallén et al., *Radio Science*, **50**(1), pp. 18-28, January 2015). In an RA sphere, the radial and tangential permittivity components can be different. Then the plasmonic singularity with its non-Rayleigh size scattering dependence generalizes into a contour in the plane defined by these two permittivity components. Also for certain combinations of positive permittivity components (one component larger, the other smaller than unity), the sphere is invisible in the static limit. In particular, the dependence of scattering by this type of spheres is to the power of +8 of the electrical size (instead of +4) which means that the static invisibility is very well retained even if the situation is dynamic but the size parameter less than unity. Furthermore, for the case of indefinite anisotropy (the permittivity components of the RA sphere are of different sign), other non-Rayleigh type size dependencies can be observed. In addition to the treatment of three-dimensional RA spheres, we will present also the anomalous scattering properties of two-dimensional RA disks (with which long circular RA cylinders with electric field excitation in the transverse plane can be modeled).