# Non-Linear Interference Mitigation Using Arrays

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*Abstract*—**Radio frequency interference obscures weak radio astronomy sources and complicates imaging the Universe; such interference from Earth-orbiting satellites has become pervasive. Prior linear approaches required estimation of the interference subspace, which can be highly dynamic and challenging. This paper develops a non-linear approach, which statistically eliminates certain strong interference signals without estimating the interference subspace across an M-element phased-array telescope. This new approach could supplement other linear processors; it does not distort beam patterns or consume spatial array degrees of freedom. The interference and desired signals can completely overlap in time and frequency with this non-linear approach. Algorithm performance with an M-element array statistically eliminates common strong interference sources but increases the thermal noise by 3dB.**

## I. INTRODUCTION

Given such powerful interference sources and such sensitive radio telescopes, the signal-to-noise ratio for an undesired satellite signal might reach +20dB, or much greater, even in a radio telescope side-lobe. Hundreds of phased array antenna elements, such as the ASKAP telescope's 188-elements, empower radio astronomers to mitigate radio frequency interference [1]. Nonetheless, side-lobe interference is difficult to model and calibrate; the signal across a phased-array feed to a parabolic dish antenna might appear to have completely random phase that evolves over time. Linear approaches estimate the interference subspace spatially to mitigate interference using projections [1]. Non-linear approaches, such as temporal or frequency excision, can help mitigate strong interference but can cause problems with missing data; this issue is most prevalent when the desired and interference signals overlap completely [2].

Another non-linear technique is intriguing because there is no need to estimate the interference's spatial signature and the interference can completely overlap the desired signal in time and frequency. This non-linear approach operates independently from all other processing, so it could supplement other phased-array linear interference mitigation techniques, which subsequently mitigate any weaker, and more challenging, interference sources [1]. This non-linear implementation has relatively low computational complexity, and does not necessarily require modifying other telescope hardware or software. Unlike other excision approaches, the new technique also works if the strong interference and the weak astronomy signal completely overlap, overcoming missing data challenges [3]. Unlike linear spatial approaches, this approach can also eliminate interference when each antenna receives a completely uncorrelated strong interference signal. Most importantly, M-element arrays using this non-linear approach can eliminate certain strong interference signals, completely, and still deliver array-gain. However, the technique effectively increases the thermal noise by 3dB.

This paper generalizes an earlier single-antenna, single-polarization, non-linear interference mitigation technique [3]. First, the performance analysis with one antenna is expounded to show that, as an ensemble average, the expected value of the desired signal phases are unperturbed by these non-linear operations. Then, using this result as a foundation, performance is generalized from a one-element antenna to an M-element array with arbitrary arrangement and polarization to highlight the performance benefits and consequences.

## II. SINGLE ANTENNA BACKGROUND

Consider a time-series signal, r[n], consisting of discrete samples which are the sum of some strong interference signals, s[n] with phase $\psi_s[n]$, the desired weak signals, d[n] with phase $\psi_d[n]$, and independent complex-valued additive white Gaussian noise, w[n], with zero-mean and variance $2\sigma^2$.

$$r[n] = s[n]e^{i\psi_s[n]} + d[n]e^{i\psi_d[n]} + w[n] \tag{1}$$

Each component might consist of several signals; there are $K_s$ strong interference signals, and $K_d$ weak desired signals, and each component has an envelope $\alpha_k[n]$ and phase $\theta_k[n]$:

$$s[n]e^{i\psi_s[n]} = \sum_{k=1}^{K_s} \alpha_k[n]e^{i\theta_k[n]} \tag{2}$$

$$d[n]e^{i\psi_d[n]} = \sum_{k=K_s+1}^{K_s+K_d} \alpha_k[n]e^{i\theta_k[n]} \tag{3}$$

Decomposing the Gaussian noise into a real-part and an imaginary-part, $w_R[n]$ and $w_I[n]$ respectively, which are independent with zero-mean and variance $\sigma^2$ each, yields (4).

$$w[n] = w_R[n] + iw_I[n] \tag{4}$$

The input signal to noise plus interference ratio equals:

$$\left(\frac{S}{N+I}\right)_{in} = \frac{d^2[n]}{2\sigma^2 + s^2[n]} \tag{5}$$

## III. POLAR EXCISION BACKGROUND

Polar excision, where polar refers to a representation in the complex-valued IQ signal plane, first decomposes the signal into a magnitude and phase portion. The algorithm transforms magnitude samples into the frequency domain, which is non-linear because the magnitude operator precedes the FFT, and excises them according to an algorithm [3]. Transforming the excised magnitude bins back into the time-domain, and recombining with the unmodified phase samples forms a complex-valued time-series signal. For processing details, see [3]. This referenced paper considers the sign of the output signal in detail, since that paper was concerned with BPSK as the weak desired signal. More generally, expounded equations for this processor are necessary to explain the output signal phase, and the statistical performance with M-element arrays.

## IV. EXPOUNDING SINGLE ANTENNA PERFORMANCE

When the strong interference presents distinct features in the transform domain, the polar excision output signal approximately equals p[n]:

$$p[n] = \left(d[n]\cos(\psi_s[n] - \psi_d[n]) + w'[n]\right)e^{i\psi_s[n]} \quad (6)$$

The noise term, w'[n], is directly related to the original noise samples:

$$w'[n] = w_R[n]\cos(\psi_s[n]) + w_I[n]\sin(\psi_s[n]) \quad (7)$$

Most noteworthy, the phase of p[n] is dependent upon the strong interference signals. When the strong interference phase varies quickly, an ensemble average approximates this practical scenario; assuming the interference phase angle, $\psi_s[n]$, is independent and uniform on $[0, 2\pi)$ yields the expected value, integrated with respect to $\psi_s[n]$ in (8).

$$E_{\psi_s}\{p[n]\} = \frac{d[n]}{2}e^{i\psi_d[n]} + \frac{w[n]}{2} \quad (8)$$

The desired signal phase angle is preserved, and the desired signal-envelope is halved. Sampling the noise only along the phase $\psi_s[n]$ halves the noise variance. The processor delivers an output signal to interference plus noise ratio:

$$\left(\frac{S}{N+I}\right)_{out} = \frac{d^2[n]}{4\sigma^2} \quad (9)$$

This scenario eliminates the strong signal interference, and penalizes the SNR by 3dB, when compared to a case with no interference.

## V. GENERALIZATION TO M-ELEMENTS

Next, the approach is generalized to M-elements. The $m^{th}$ array element signal equals:

$$r_m[n] = s_m[n]e^{i\psi_{s,m}[n]} + d_m[n]e^{i\psi_{d,m}[n]} + w_m[n] \quad (10)$$

The noise at each element is complex-valued and independent with zero-mean and variance $2\sigma^2$. Each signal component now has an additional term that models the phase angle at each array element versus time, $\phi_{k,m}[n]$. One signal could consist of multiple components to overcome the quasi-monochromatic modeling assumption, and the signal magnitudes can vary between antennas or sites:

$$s_m[n]e^{i\psi_{s,m}[n]} = \sum_{k=1}^{K_s}\alpha_{k,m}[n]e^{i\theta_k[n]}e^{i\phi_{k,m}[n]} \quad (11)$$

$$d_m[n]e^{i\psi_{d,m}[n]} = \sum_{k=K_s+1}^{K_s+K_d}\alpha_{k,m}[n]e^{i\theta_k[n]}e^{i\phi_{k,m}[n]} \quad (12)$$

Each element will undergo an independent polar excision:

$$p_m[n] = \left(d_m[n]\cos(\psi_{s,m}[n] - \psi_{d,m}[n])\right)e^{i\psi_{s,m}[n]} + w''_m[n] \quad (13)$$

$$w''_m[n] = w'_m[n]e^{i\psi_{s,m}[n]} \quad (14)$$

A processor will then compute the intensity toward a particular desired source component in the astronomy image. A simple processor might apply (15) where an unknown, but constant, receiver phase offset $\Delta$ exists.

$$y_o[k,n] = \left|\sum_{m=1}^{M}p_m[n]e^{-i(\phi_{k,m}[n]+\Delta)}\right|^2 \qquad k \geq K_s + 1 \quad (15)$$

When $\psi_{s,m}[n]$ is independent across the m-array elements, then:

$$E\{y_o[k,n]\} = \left|\sum_{m=1}^{M}\left(\frac{d_m[n]}{2}e^{i\psi_{d,m}[n]} + \frac{w_m[n]}{2}\right)e^{-i(\phi_{k,m}[n]+\Delta)}\right|^2 \quad (16)$$

Assume the other points in the desired astronomy scene are spatially orthogonal to this point and that the desired signal has the same amplitude at each antenna element. Neither assumption is necessary, but these assumptions simplify the signal to interference plus noise ratio expression to illustrate the performance characteristics in (17).

$$\left(\frac{S_k}{N+I}\right)_{final} = \frac{M\alpha_k^2[n]}{4\sigma^2} \qquad k \geq K_s + 1 \quad (17)$$

In summary, the output signal to interference plus noise ratio improves with the number of elements, M. The strong interference vanishes, albeit the thermal noise increases by 3dB.

## REFERENCES


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