Dispersion Relation for Cylindrical FDTD Grids

Mohammed F. Hadi*[1,2,3], Atef Z. Elsherbeni[2], Melinda J. Piket-May[3], and Samir F. Mahmoud[1]

[1] Electrical Engineering, Kuwait University
[2] Electrical Engineering and Computer Science, Colorado School of Mines
[3] Electrical, Computer and Energy Engineering, University of Colorado at Boulder

The body-of-revolution finite-difference time-domain (BOR-FDTD) method is based on a compact two-dimensional grid within the ρ-z plane for modeling rotationally symmetrical structures where the ϕ-dependence (third dimension) is explicitly predetermined before Maxwell's equations are converted to finite differences. As an example, the update equation for the ρ-component of the electric field is given by

$$E_\rho|_{r,\kappa}^{n+\frac{1}{2}} = E_\rho|_{r,\kappa}^{n-\frac{1}{2}} - \frac{H_\phi|_{r,\kappa+\frac{1}{2}}^{n} - H_\phi|_{r,\kappa-\frac{1}{2}}^{n}}{\epsilon\Delta z/\Delta t} + \frac{mH_z|_{r,\kappa}^{n}}{(r\Delta\rho)\epsilon/\Delta t}$$

where r, κ, n are spatial and temporal indices, $\Delta\rho, \Delta z, \Delta t$ are the corresponding discrete steps and m is a predetermined rotational mode.

Practical and efficient implementation of BOR-FDTD requires fine tuning the associated modeling tools (e.g., discrete step selection, absorbing boundary conditions and source initiation) to maintain amplitude and phase errors control. Such fine tuning is based on the behavior of the underlying grid structure which is governed by its discrete dispersion relation. Whereas the dispersion relations for the various FDTD algorithms that are based on Cartesian grids are well documented in the literature, corresponding relations for curvilinear grids have not been fully developed. The limited attempts in the literature for this purpose relied on the same methodology used for Cartesian grids which resulted in dispersion relations that are usable only at large distances from the axis of rotation where radial waves approach plane wave behavior.

To derive the proper dispersion relation for the BOR-FDTD method, the initial harmonic waves that need to be injected in the discrete update equations should have the form of Hankel function cylindrical waves instead of plane wave harmonics. Further, different field components require certain combinations of the Hankel function and its derivative to satisfy Maxwell's equations within cylindrical coordinates. This procedure will be detailed during the conference, along with validating examples and a quick discussion of numerical dispersion impact on BOR-FDTD simulations.