Use of Li's improved Levin Method for Highly Oscilatory Reflector Antenna Diffraction Kernel

Arthur Densmore and Yahya Rahmat-Samii
Electrical Engineering Department
University of California, Los Angeles
Los Angeles, California, United States
adensmore@ucla.edu and rahmat@ee.ucla.edu

I. ABSTRACT

Li's improved Levin method provides dramatic acceleration of brute-force physical optics applied to a parabolic reflector of 240 wavelengths, especially in the far-out sidelobe regions. The method involves the use of an accelerating approximation with a delaminating quadrature provided by Li, sampling at Chebyshev-Lobatto nodes, makes use of the Chebyshev differential matrix, and truncated singular value decomposition.

Levin method [1], [2] is a relatively new technique for fast and accurate integration of highly oscillatory functions. The Levin method turns a highly oscillatory numerical integral into an equivalent partial differential equation that is solved using a matrix collocation method requiring only sparse sampling of the integrand, including the derivative of its exponential argument. The original Levin method required a minimum phase derivative over the integration domain for convergence and thereby has difficulty calculating the main beam direction. Li [3], [4] tackles that problem by revealing that the rank of the target matrix allows application of the truncated singular value decomposition (TSVD) method to solve the matrix system with accuracy in spite of being ill-conditioned, using a Chebyshev differential matrix on Chebyshev-Lobatto nodes. The examples provided in Li's papers are convex scattering surfaces; whereas, in this paper we revisit the technique and apply it to concave parabolic reflector with emphasis to both main beam and far-out sidelobe regions.

Li's revision solves several equivalent partial differential equations along strips parallel to the x-axis; then each end point value is used to solve two last partial differential equations parallel to the y-axis – Those two last solutions become the surface integral result below:

$$I = \int_c^d \left[\int_a^b f\left(x,y\right) \exp\left[jg\left(x,y\right) \right] dx \right] dy \qquad (1)$$

by applying the following accelerating approximation:

$$I \sim \int_c^d \left[\int_a^b \frac{d}{dx} \left\{ p\left(x,y\right) \exp\left[jg\left(x,y\right) \right] \right\} dx \right] dy, \qquad (2)$$

where $p_x\left(x,y\right) + jg_x\left(x,y\right) p\left(x,y\right) = f\left(x,y\right)$. (3)

$p\left(x,y\right)$ is solved along strips of x by matrix collocation. Typical Physical Optics (PO) integrals resemble (1) [5].

This procedure involves solving N+1 matrices of size N+1, that represent the first dimension of integration (e.g., *x*-axis), then subsequently two more systems each of matrix size N+1, are solve along the orthogonal axis using the ends points of the results obtained along the former axis.

This method uses Chebyshev polynomial basis functions to substantially speed up the computations. Chebyshev polynomials converge faster than other polynomials. The acceleration is due to the use of the Chebyshev differential matrix, requiring that the function consist of Chebyshev basis functions sampled at the Chebyshev-Lobatto nodes.

The former Levin method solves a single matrix system of dimension $(N+1)^2$, instead of Li's much smaller N+1 dimension, and involves additional arithmetic regarding basis function expansion that the new method avoids, that helps it run even faster. Representative large reflector antenna examples will be presented to demonstrate the utility or the method for the far-field pattern computations.

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