A Classification Framework for Methods of Uncertainty Quantification in Computational Electromagnetics

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Growing interest in developing efficient and accurate computational methods to quantify uncertainties in electromagnetic fields is driven by limitations in materials characterization techniques and manufacturing processes – both of which lead to variability and uncertainty in the physical and electromagnetic properties of materials and devices. A variety of computational electromagnetic methods have been proposed for estimating the mean and variance of fields in the presence of uncertain materials properties and dimensions. These techniques differ extensively in their computational complexity and accuracy. They also tend to target very specific application problems and make use of opaque terminology. There is a growing need to classify various methods based on the underlying mathematical techniques or tools that they incorporate and to clarify and standardize the terminology.

All uncertainty quantification methods treat the fields or derived quantities of interest such as S-parameters as stochastic functions of multiple random variables representing the uncertain properties. Uncertainty quantification is viewed as the mathematical problem of evaluating the mean and variance of functions of multiple random variables. In this paper, we present a classification framework to organize current uncertainty quantification methods into well-defined categories based on the approach used to represent variation in the electromagnetic fields and the mathematical techniques employed to estimate the mean and variance integrals of stochastic functions. For example, a high-level differentiating characteristic of existing methods is whether the functions are represented as a weighted sum of polynomial basis functions with unknown coefficients or whether functions are evaluated at sample points drawn from the assumed distribution of the uncertain parameters. Techniques based on basis function representations are further differentiated by the method used to estimate the unknown coefficients.

Our proposed framework elucidates the relationship between methods and clearly highlights their similarities and differences, including computational and performance trade-offs. It also simplifies the selection of a particular method for a specific problem.